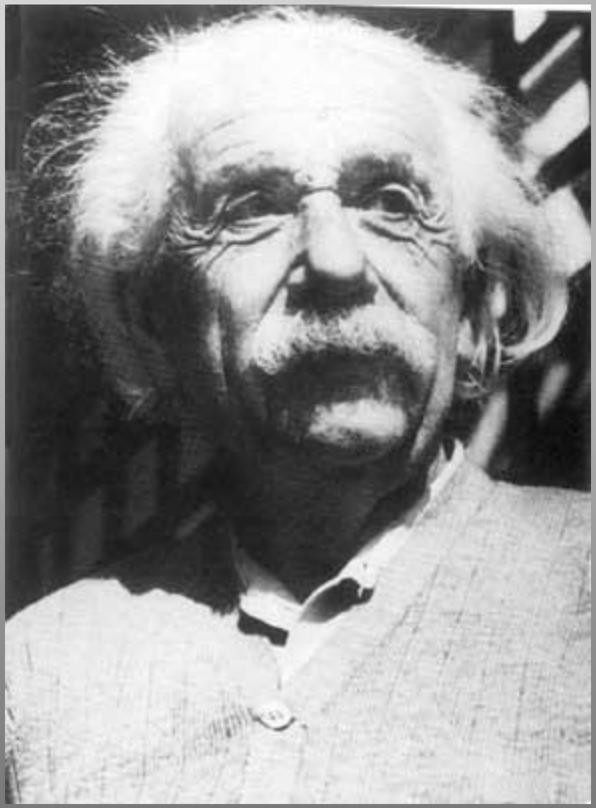
Chapter 2
Special Theory of Relativity



Albert Einstein (1879 – 1955) Nobel, 1921

Einstein's Miraculous Year (1905)

- All papers published in the German journal ***Annalen der Physik.***
- Completed Ph.D. Dissertation, University of Zurich, July 20, 1905: *A New Determination of Molecular Dimensions.* Published August 19, 1905.

1. ***On the Motion of Small Particles Suspended in Liquids at Rest Required by the Molecular-Kinetic Theory of Heat*** (*Annalen der Physik* 17 [1905]: 549-560.)
2. ***On the Electrodynamics of Moving Bodies*** (*Annalen der Physik* 17 [1905]: 891-921.)
3. ***On a Heuristic Point of View Concerning the Production and Transformation of Light*** (*Annalen der Physik* 17 [1905]: 132-148.)

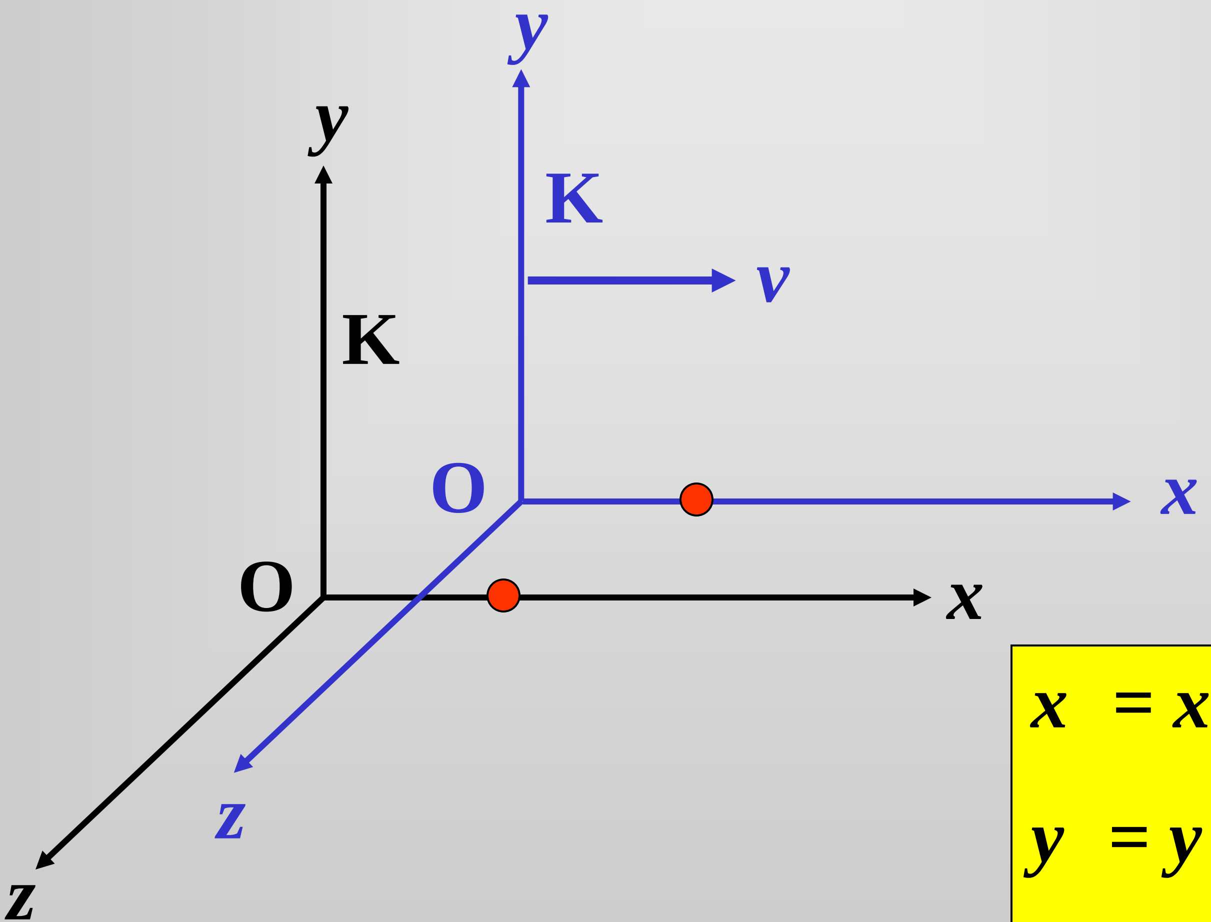
Introduction

- Newtonian principle of relativity
- Galilean invariance

Galilean-Newtonian Relativity

- Galilean-Newtonian relativity is known as a "classical" theory.
- Einstein's special theory of relativity is known as a "modern" theory.

Galilean Transformation



$x = x - vt$

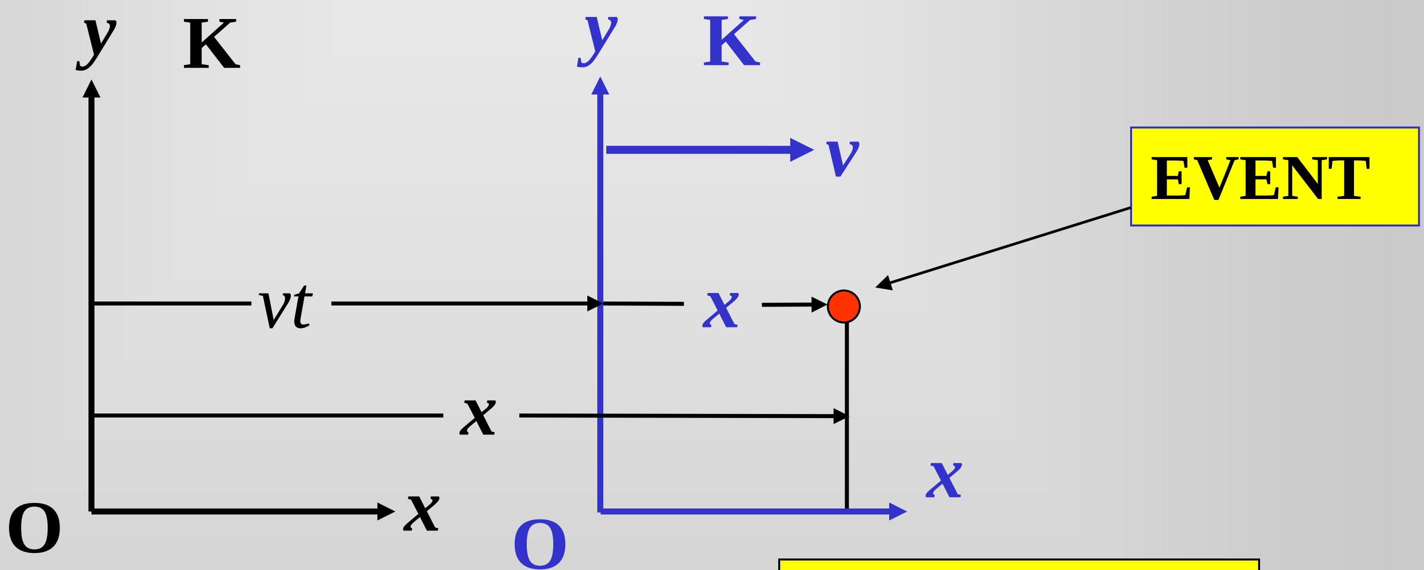
$y = y$

$z = z$

$t = t$

Time is absolute

Galilean Transformation



$$x = x - vt$$

$$y = y$$

$$z = z$$

$$t = t$$

Time is absolute

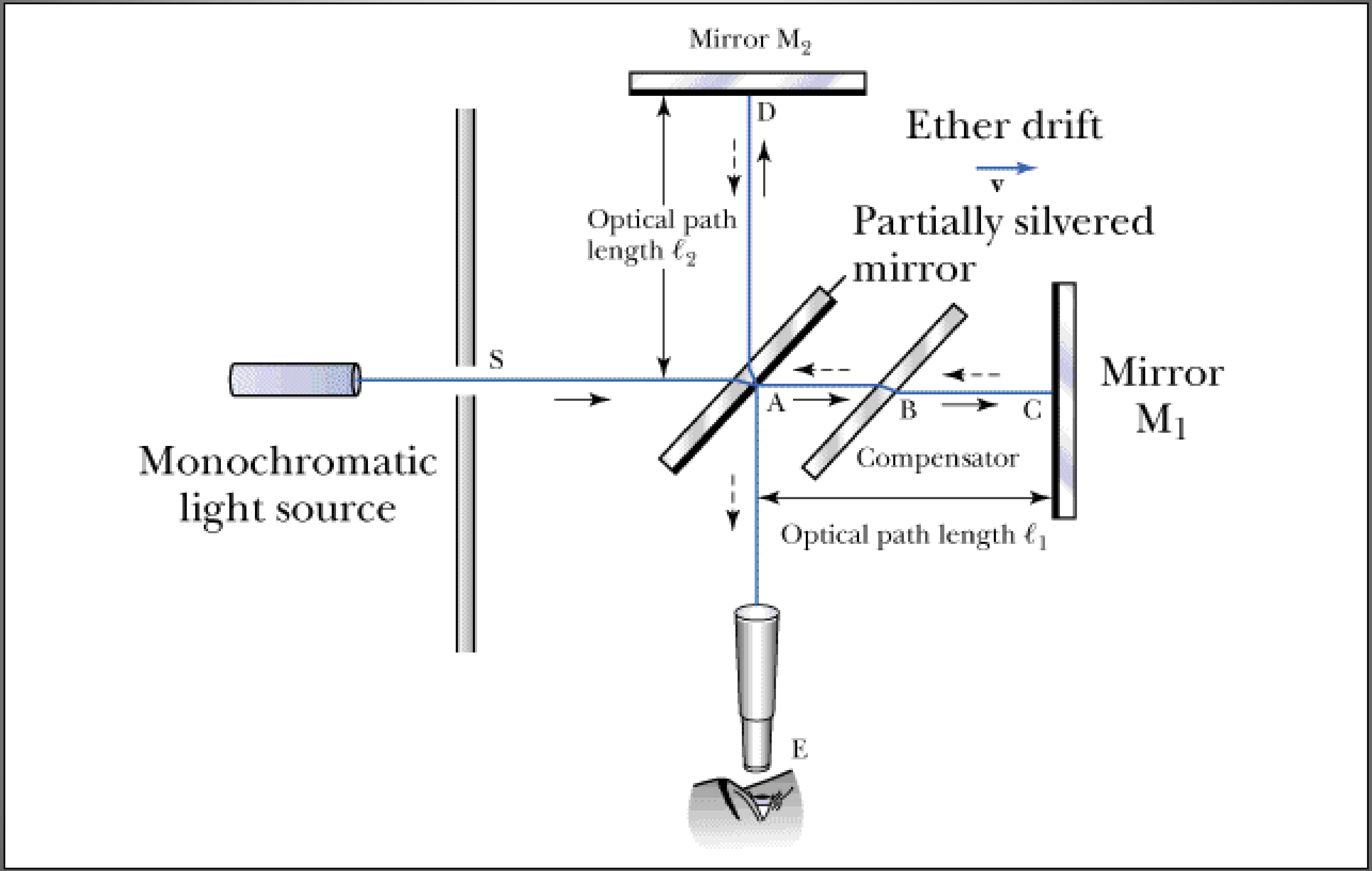
2.1 Historical Perspective

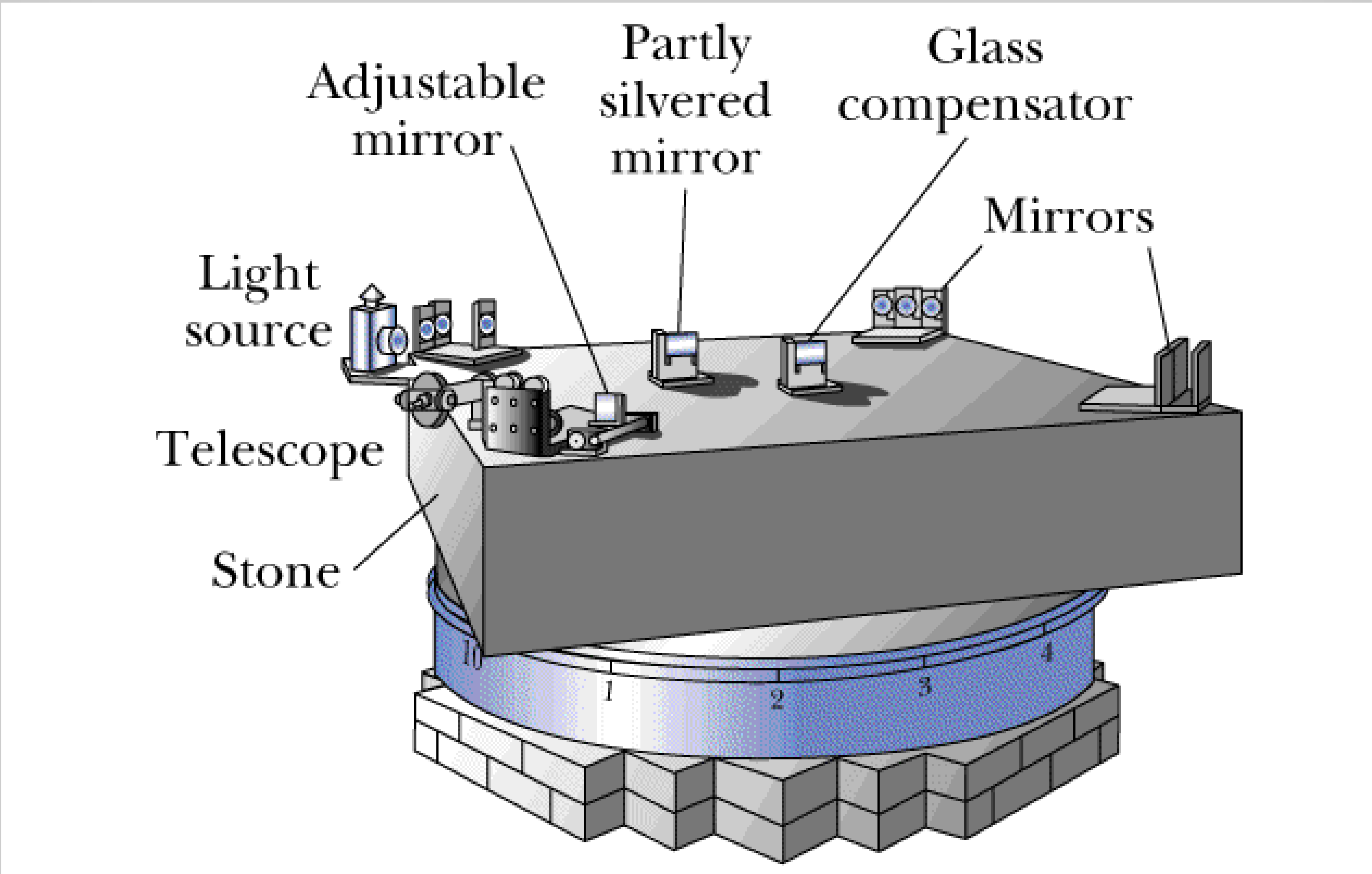
- Light is a wave.
- Waves require a medium through which to propagate.
- Medium as called the “ether.” (from the Greek *aither*, meaning upper air)
- Maxwell’s equations assume that light obeys the Newtonian-Galilean transformation.

2.2 The Michelson-Morley Experiment

- Experiment designed to measure small changes in the speed of light was performed by Albert A. Michelson (1852 – 1931, Nobel) and Edward W. Morley (1838 – 1923).
- Used an optical instrument called an *interferometer* that Michelson invented.
- Device was to detect the presence of the ether.
- Outcome of the experiment was negative, thus contradicting the ether hypothesis.
- **A.A. Michelson and E.W. Morley, *American Journal of Science*, 134 – 333, 1887)**

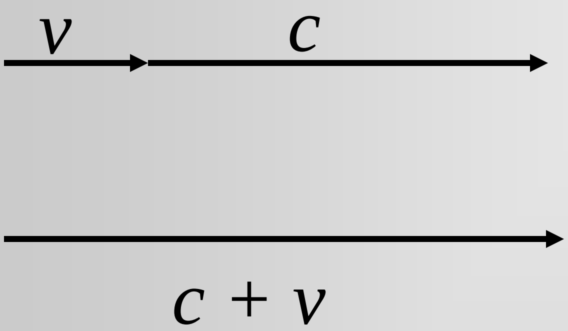
Michelson-Morley Experiment



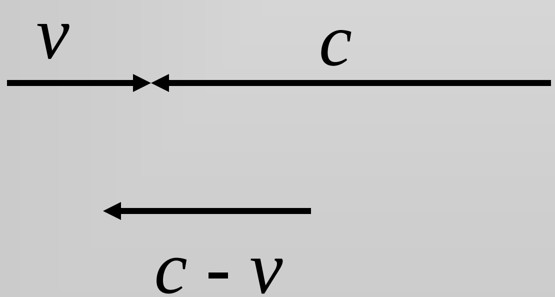


Adjustable mirror
Partly silvered mirror
Glass compensator
Mirrors
Light source
Telescope
Stone
10
1
2
3
4

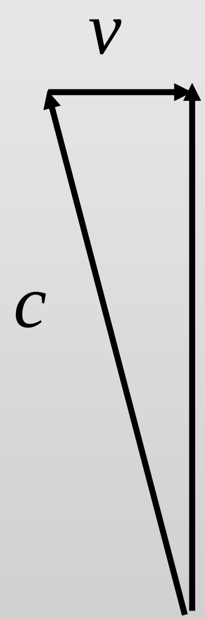
The Ether



"Upwind"

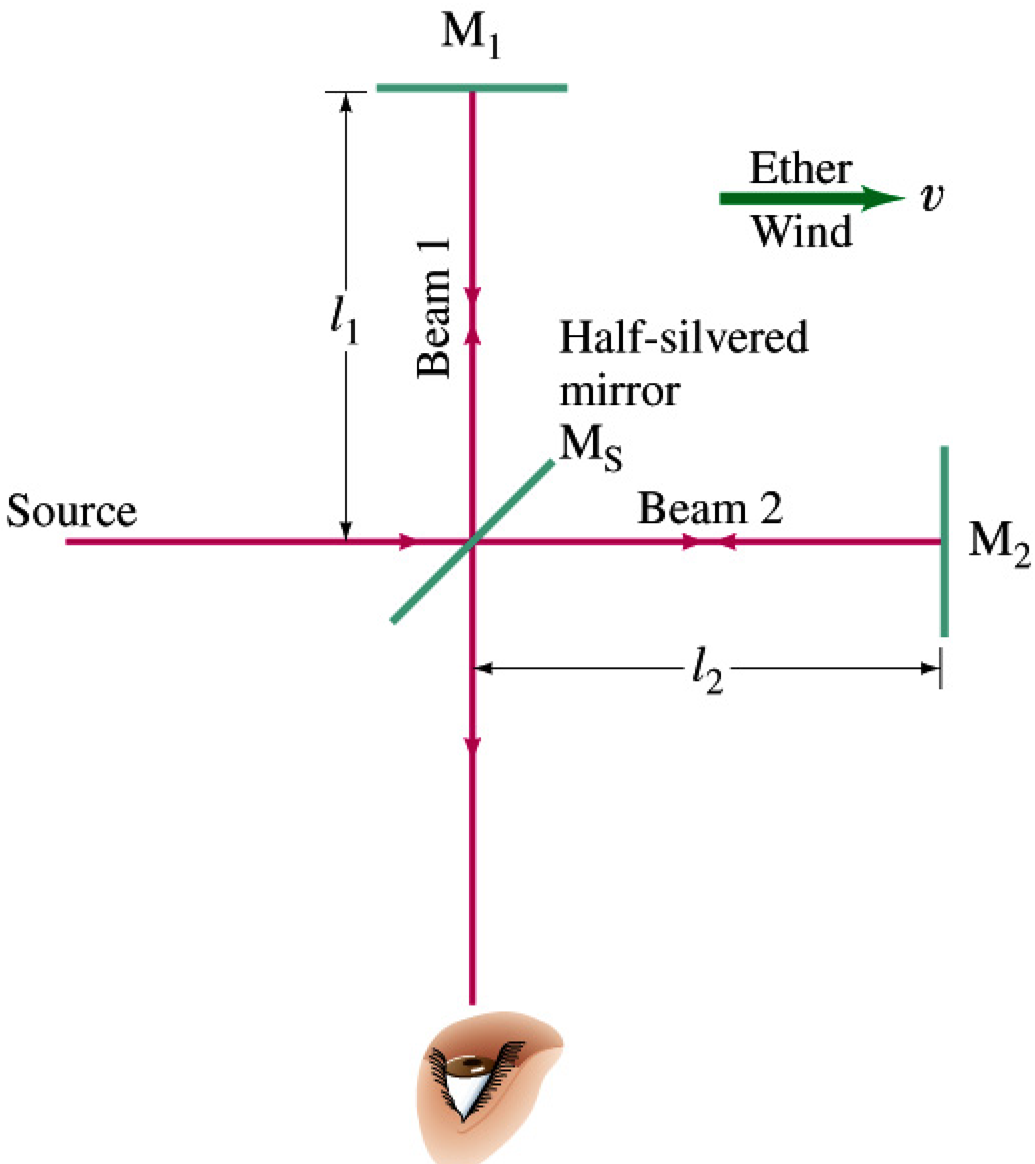


"Downwind"



$$v = (c^2 - v^2)^{1/2}$$

Perpendicular to "wind."


M1
Ether
Wind
v
Beam 1
l1
Half-silvered
mirror
MS
Source
Beam 2
M2
l2

2.3 Einstein’s Postulates

Big problem at the turn of the century:

1. Michelson-Morley showed that the Galilean transformation did not hold for Maxwell’s equation.
2. Maxwell’s equations could not be wrong.
3. Galilean transformation did hold for the laws of mechanics.
4. Einstein proposed a solution.

Inertial Reference Frame

An inertial reference frame is one

- in which no accelerations are observed in the absence of external forces.
- that is not accelerating.
- Newton's laws hold in all inertial reference frames.

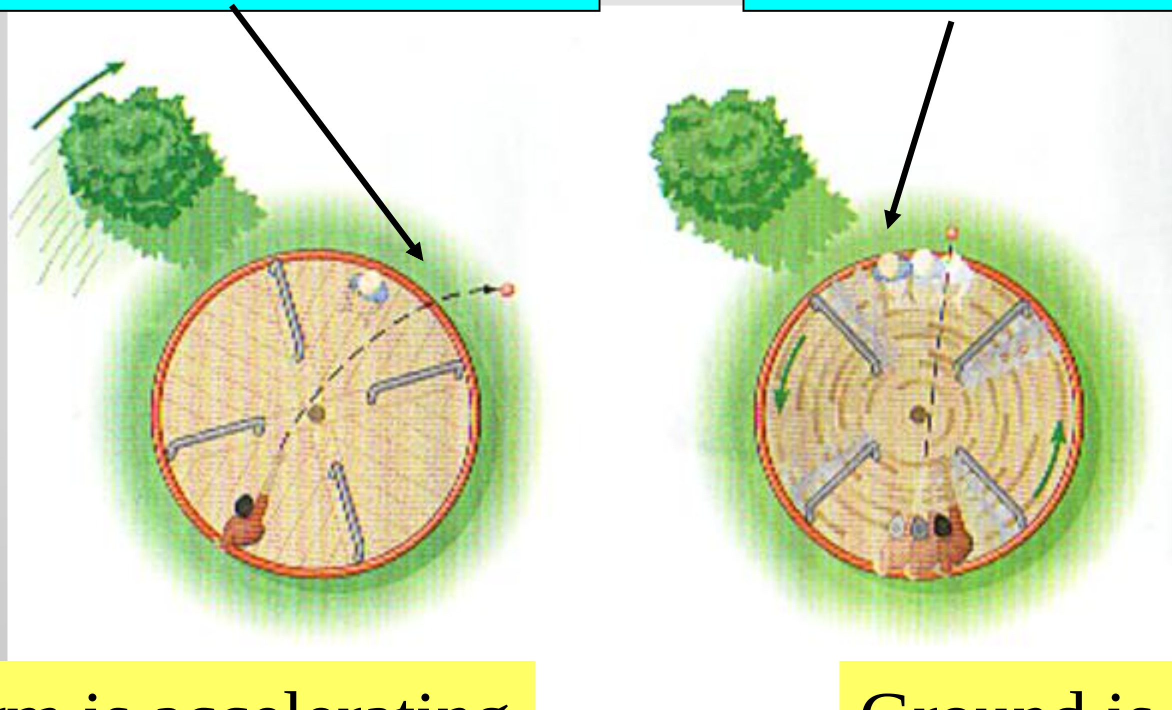
Noninertial Reference Frame

- A noninertial reference frame is one that is accelerating with respect to an inertial reference frame.
- In a noninertial reference frame, bodies have accelerations in the absence of applied forces.

Reference Frames

Platform at rest, tree moving—ball is seen by <u>observers on platform</u> as being deflected, but no force acts on it. Violation of Newton's second law.

Platform moving. Observer on the ground (inertial frame) sees ball move in a straight line, but sees the catcher move away.



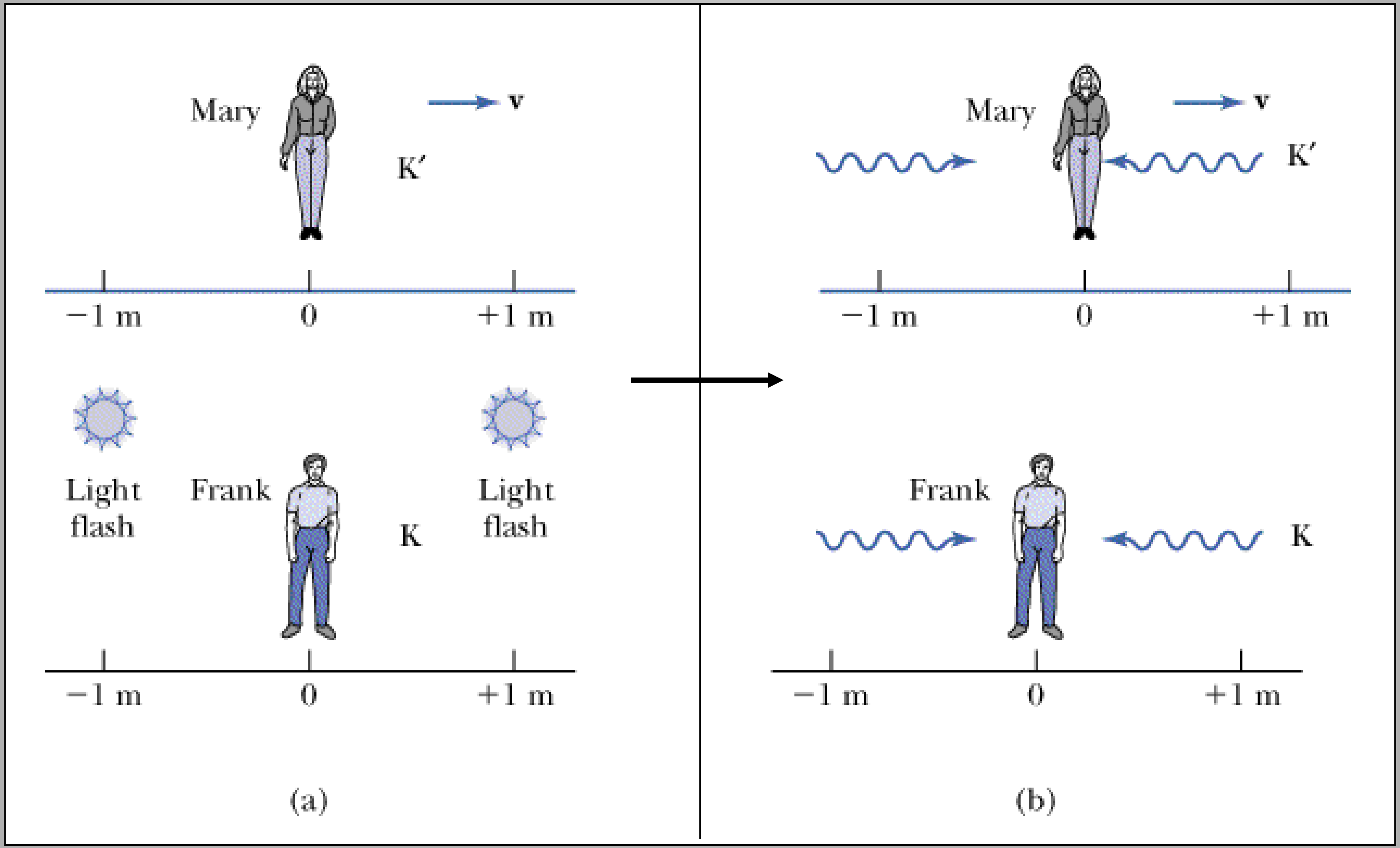
Platform is accelerating noninertial frame

Ground is the inertial frame

The Earth

- In many cases the Earth's surface can be considered an inertial reference frame, even though strictly speaking it is not.
- For small scale phenomena the Earth is approximately an inertial reference frame.

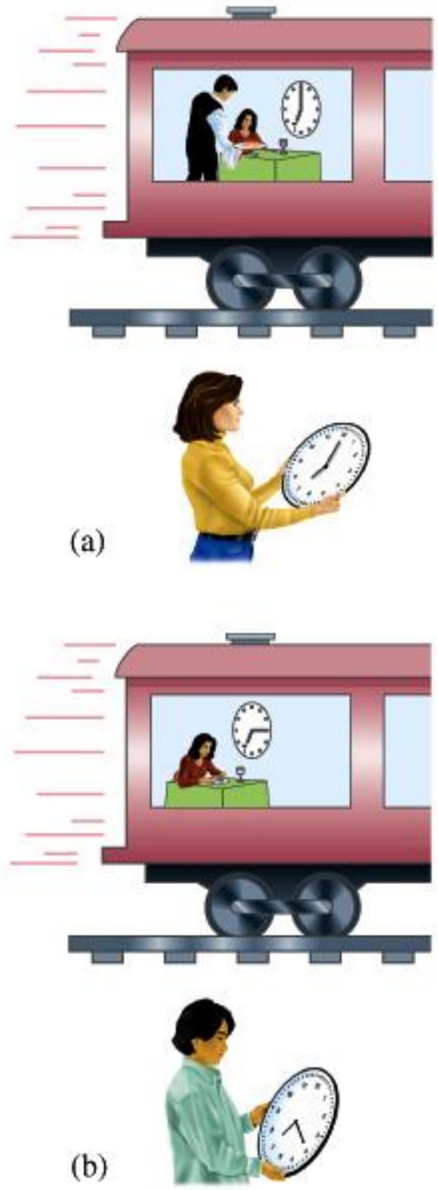
Problem of Simultaneity



Conclusion

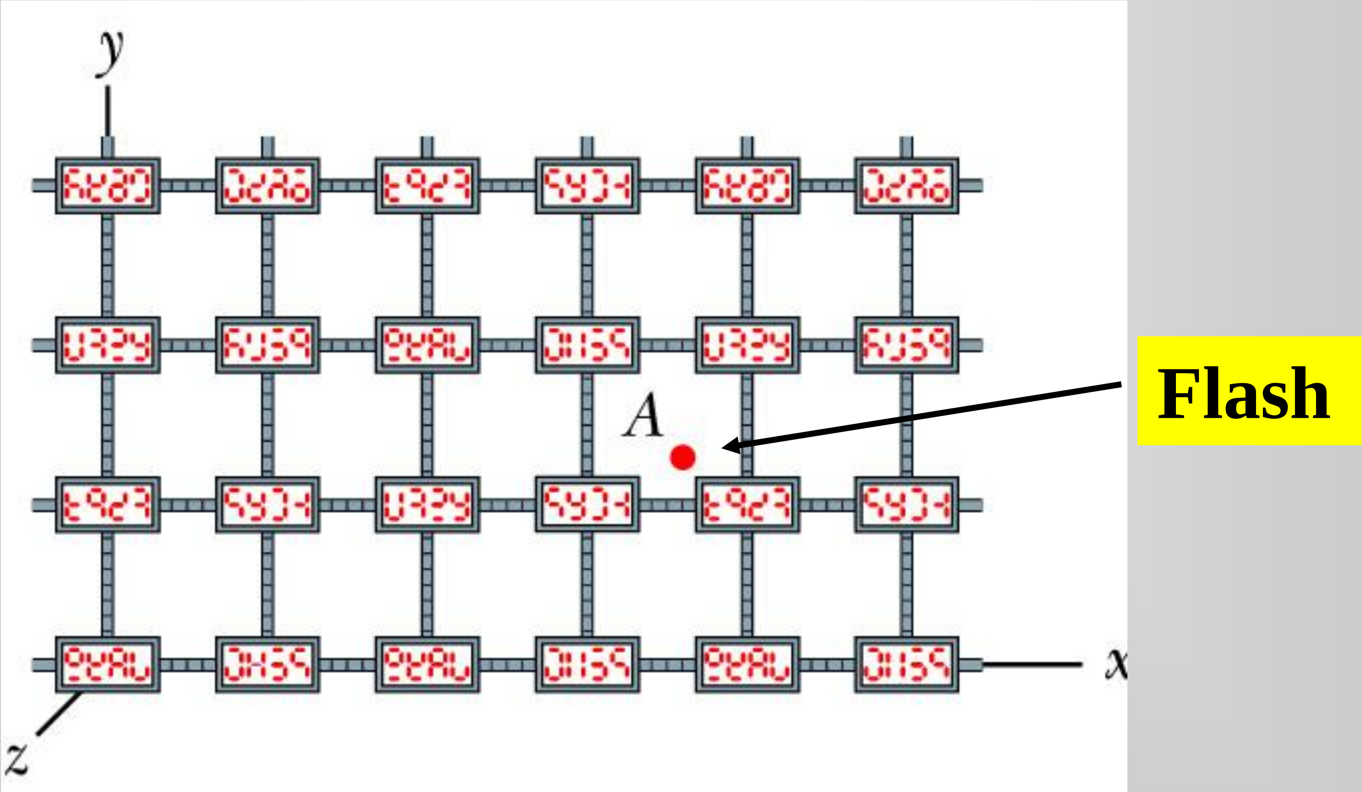
- An event in a given system must be specified by stating both its space and time coordinates.
- Result: The principle of simultaneity.

Four- Dimensional Spacetime



Spacetime Coordinates

(x, y, z, t)



x = 3.7 rods, y = 1.2 rods, z = 0 rods, t = whatever time appears on the clock nearest to A at the time of the flash.

Principle of Simultaneity

Two events that are simultaneous in one reference frame (K) are not necessarily simultaneous in another reference frame (K) moving with respect to the first frame.

Recall that in the Galilean transformation time is considered absolute regardless of the relative motion of the reference inertial reference frames.

Einstein's Postulates

1. The Principle of Relativity
2. The constancy of the speed of light.

The Principle of Relativity

All the *laws* of physics are the *same* in all inertial systems.* There is no way to detect absolute motion, and no preferred inertial system exists.

*Particular quantities (velocity, momentum, kinetic energy, …) have different values in different inertial reference frames, but the laws of physics (conservation of energy and momentum, …) are the same.

The Constancy of the Speed of Light

Observers in all inertial systems measure the same value for the speed of light in a vacuum. ($c = 2.9979 \times 10^8$ m/s)

The Ultimate Speed

- The speed of light has been defined to be exactly:

$$c = 299\,792\,458 \text{ m/s}$$

- Light travels at this ultimate speed, as do any massless particles.
- No entity that carries energy or information can exceed this speed limit.
- No particle that does have a mass, can actually reach *c*.
- Electrons have been accelerated to at least 0.999 999 999 95 times the speed of light—still less than *c*.

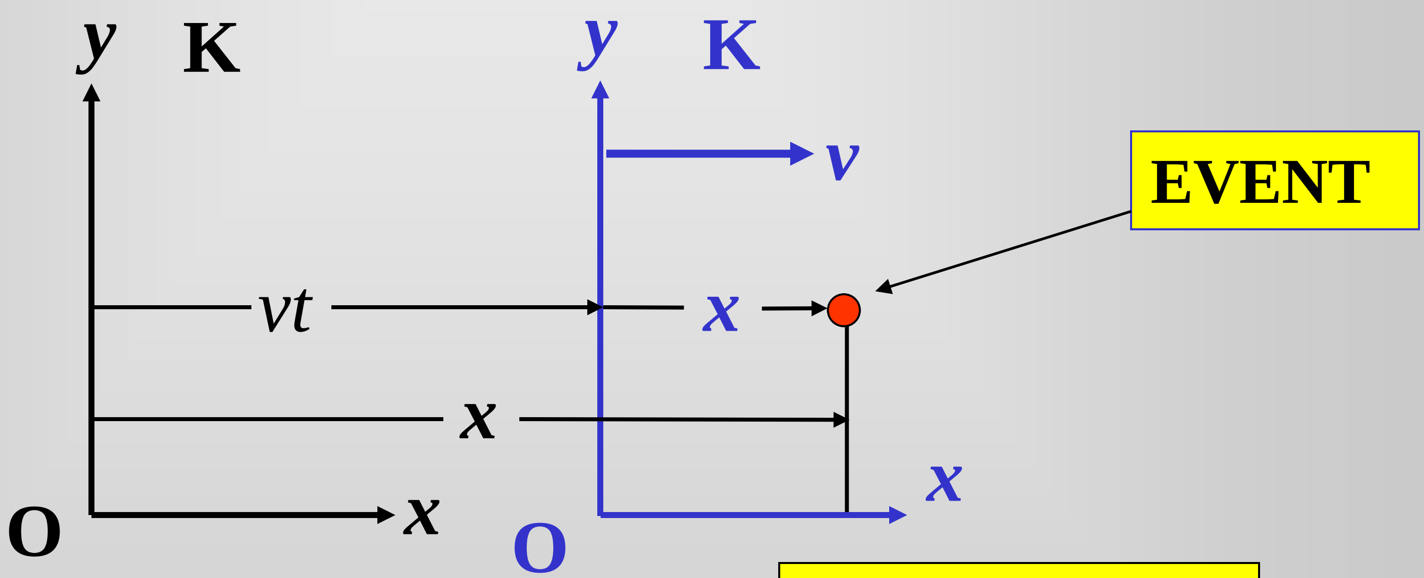
2.4 The Lorentz Transformation

- Einstein needed to find a new transformation (the old one being the Galilean transformation).
- It must fit both the laws of mechanics and Maxwell's electrodynamic equations.
- It must allow time to be relative.

Historical Note

- The Lorentz transformation was derived in 1890 by Hendrik A. Lorentz (1853 – 1928, Nobel 1902) to explain the null result in the Michelson-Morely experiment.
- He proposed that one of the arms of the interferometer would contract by a factor of $(1-v^2/c^2)^{1/2}$ explaining the null result.
- Only basis was to fit the experimental data.

Galilean Transformation



$$x \;\; = x - vt$$

$$y \;\; = y$$

$$z \;\; = z$$

$$t \;\; = t$$

Time is absolute

Lorentz Transformation Equations

$$x = \frac{x - vt}{\sqrt{1-\beta^2}}$$

$$y = y$$

$$z = z$$

$$t = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\beta^2}}$$

where

$\beta = v/c$

Lorentz Transformation Equations

$$x = \gamma(x - vt)$$

$$y = y$$

$$z = z$$

$$t = \gamma\left(t - \frac{vx}{c^2}\right)$$

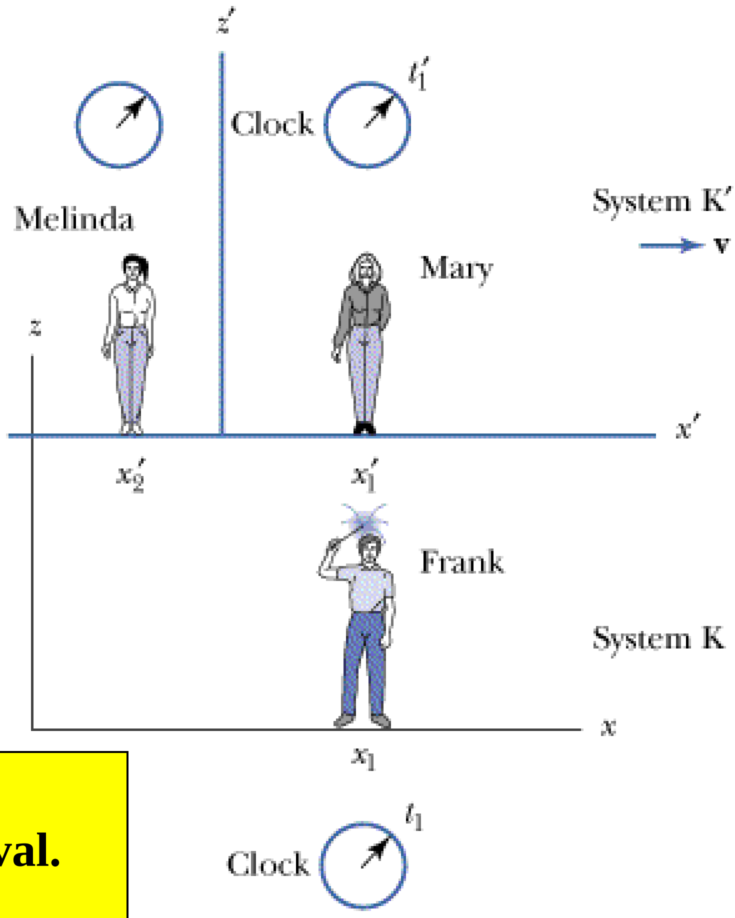
where $\gamma =$

$$\sqrt{1-\beta^2}$$

where

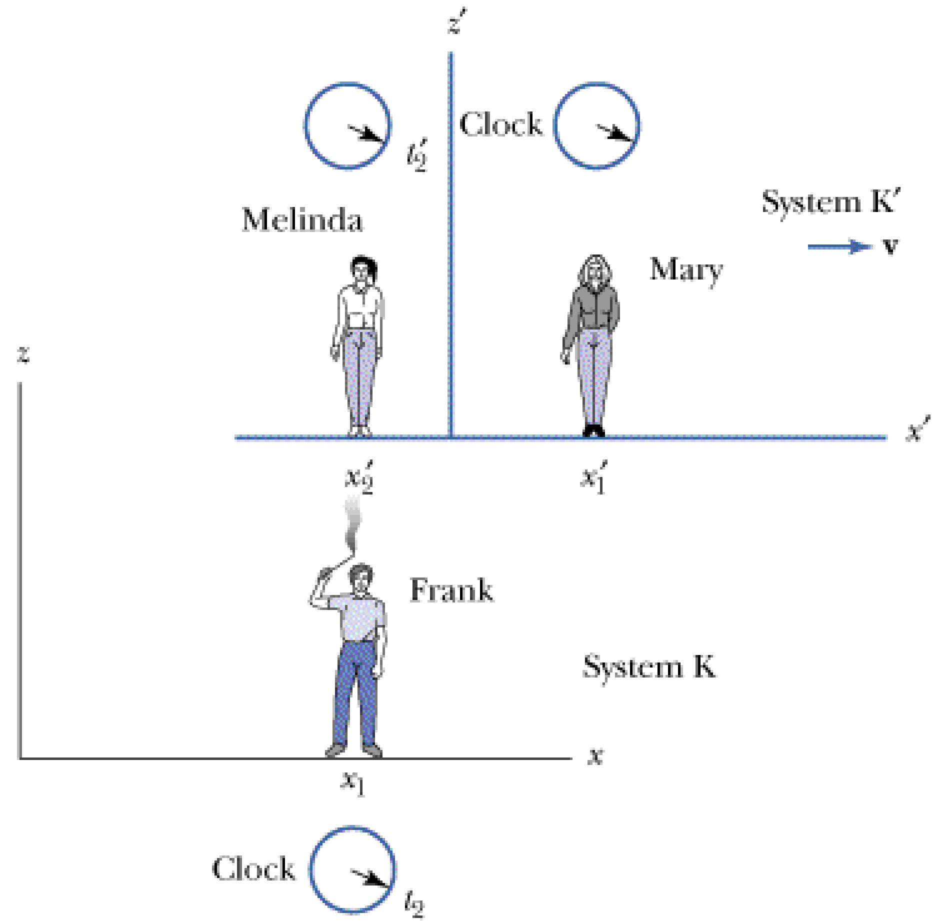
$$\beta = v/c$$

2.5 Time Dilation and Length Contraction



(a)

Frank measures proper time interval. Why?


z′
Clock
t_2'
Melinda
Mary
System K′
v
z
x′
x_2'
x_1'
Frank
System K
x
x_1
Clock
t_2
(b)

Time Dilation

Time where clock is **moving** relative to the observer.

Time where clock is **at rest** relative to the observer. **Proper time.**

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

Clocks moving relative to an observer are measured by that observer to run more slowly, as compared to the clock at rest.

Time Dilation

$$T = \gamma T_0$$

Where: $T = \Delta t$ **and** $T_0 = \Delta t_0$

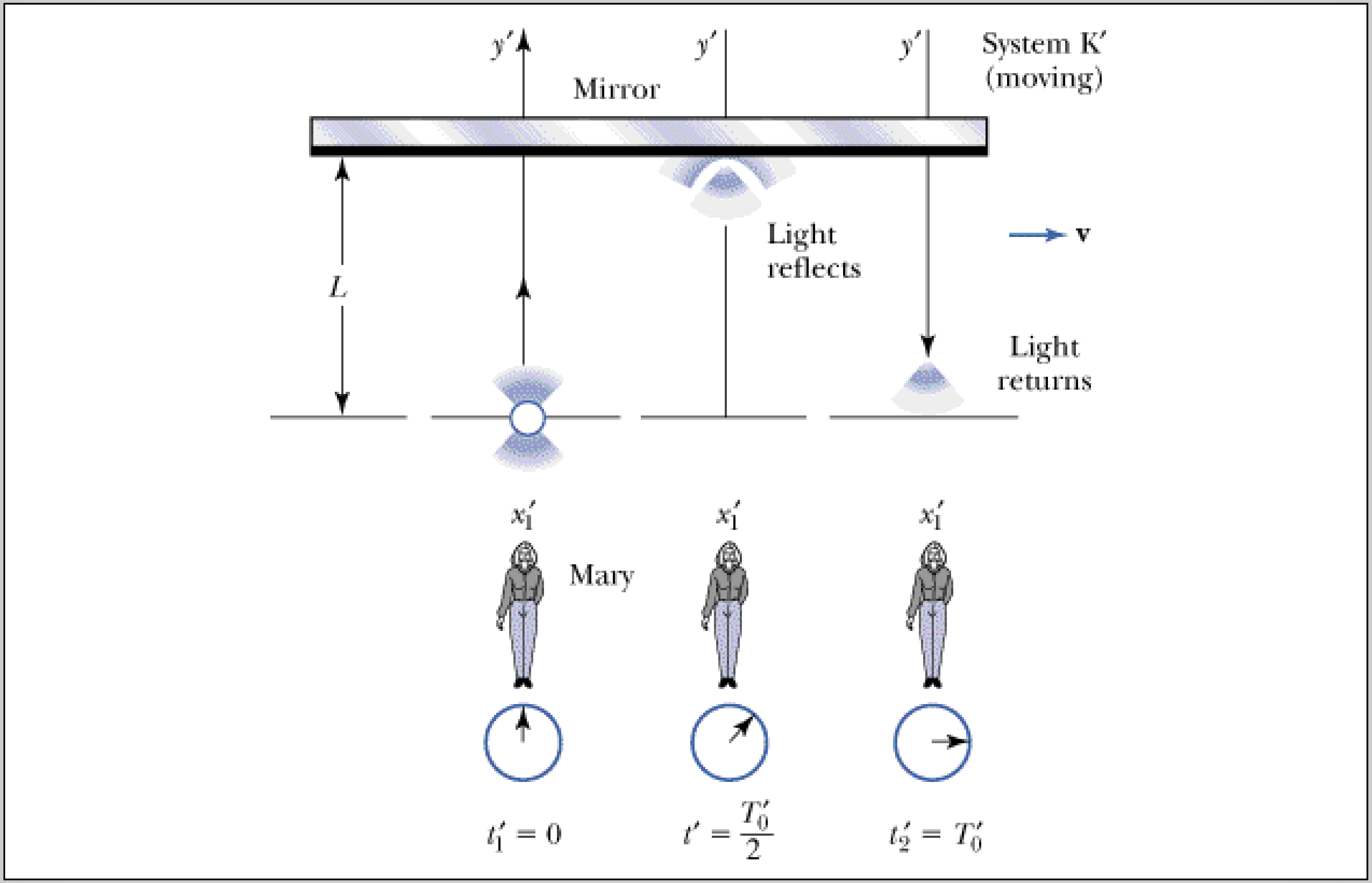
Example 2.1

Show that Frank in the fixed system will also determine the time dilation result by having the sparkler be at rest in the system K'.

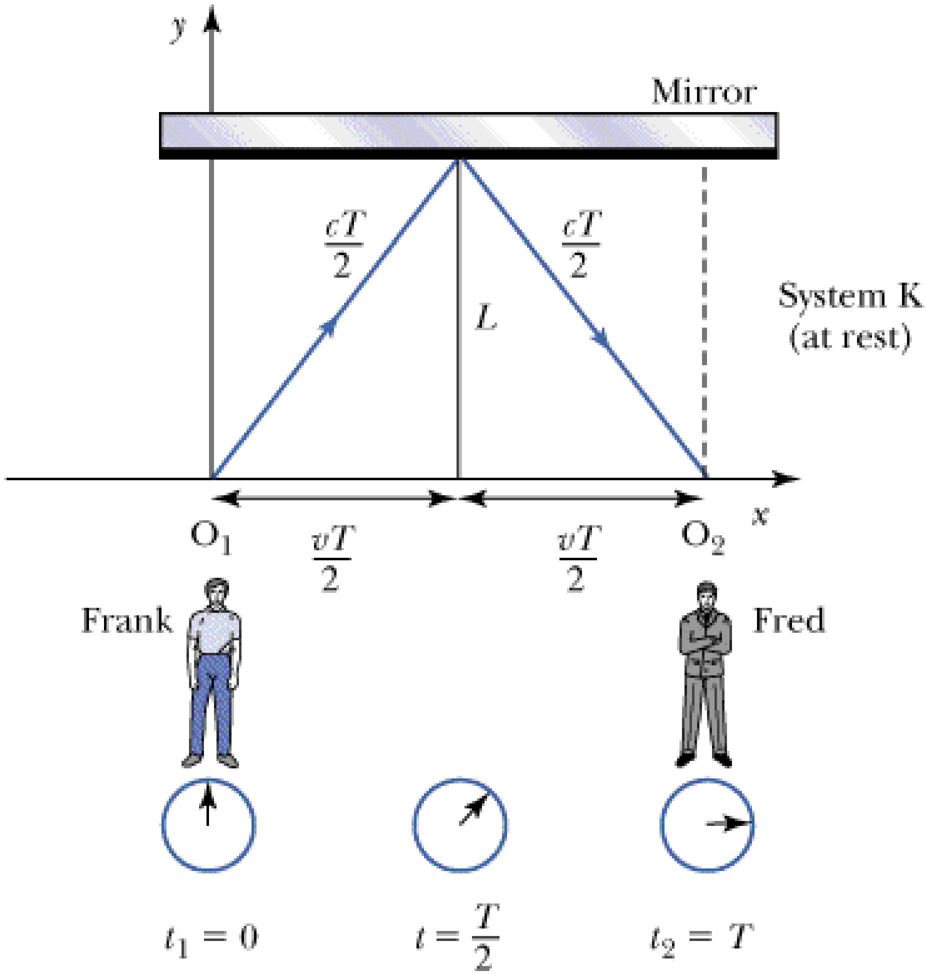
Time Dilation

$$T = \gamma T_0$$

For clock in the K frame

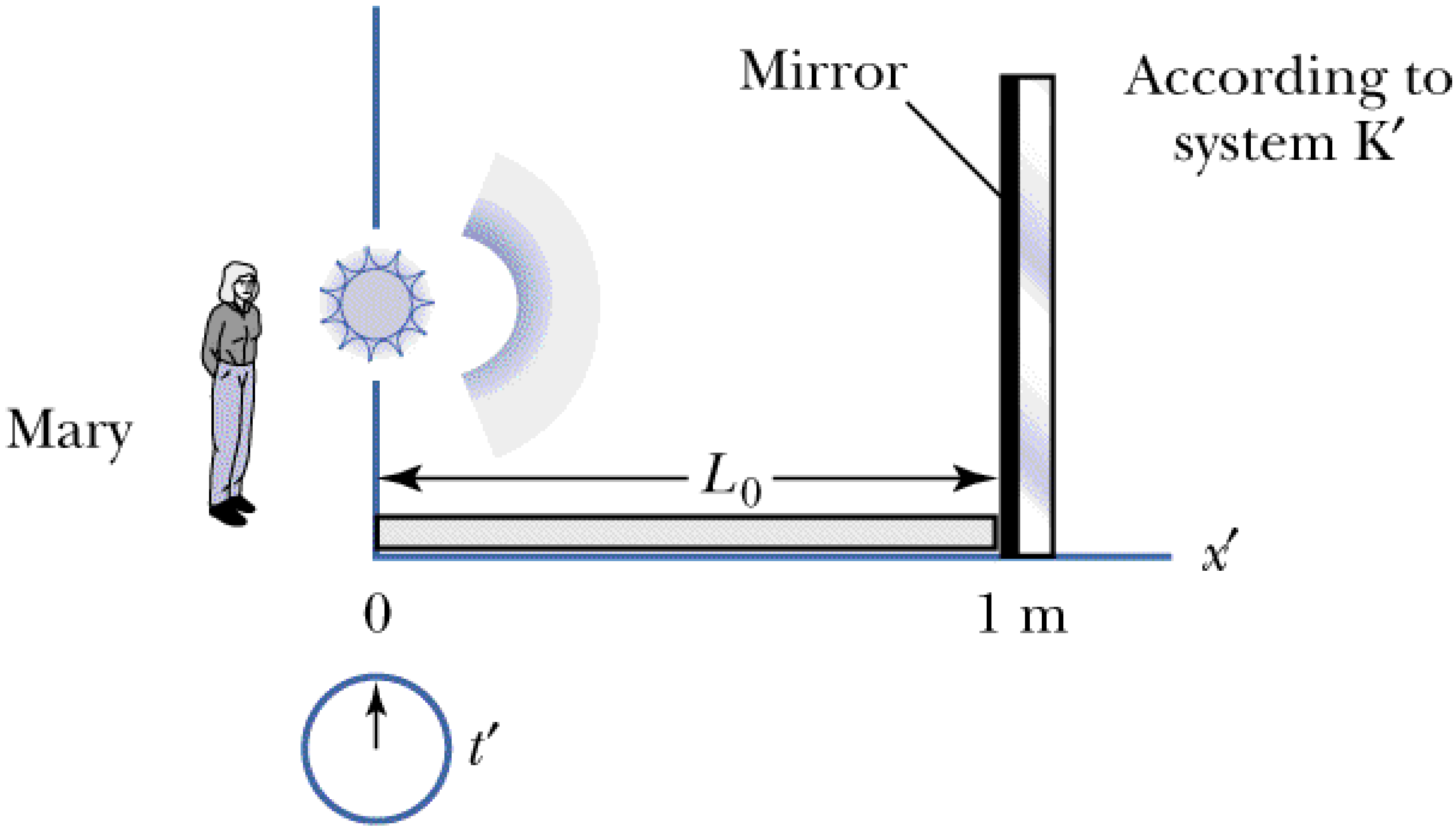


y′
y′
y′
System K′
(moving)
Mirror
Light
reflects
v
L
Light
returns
x_1'
x_1'
x_1'
Mary
$t_1' = 0$
$t' = \frac{T_0'}{2}$
$t_2' = T_0'$

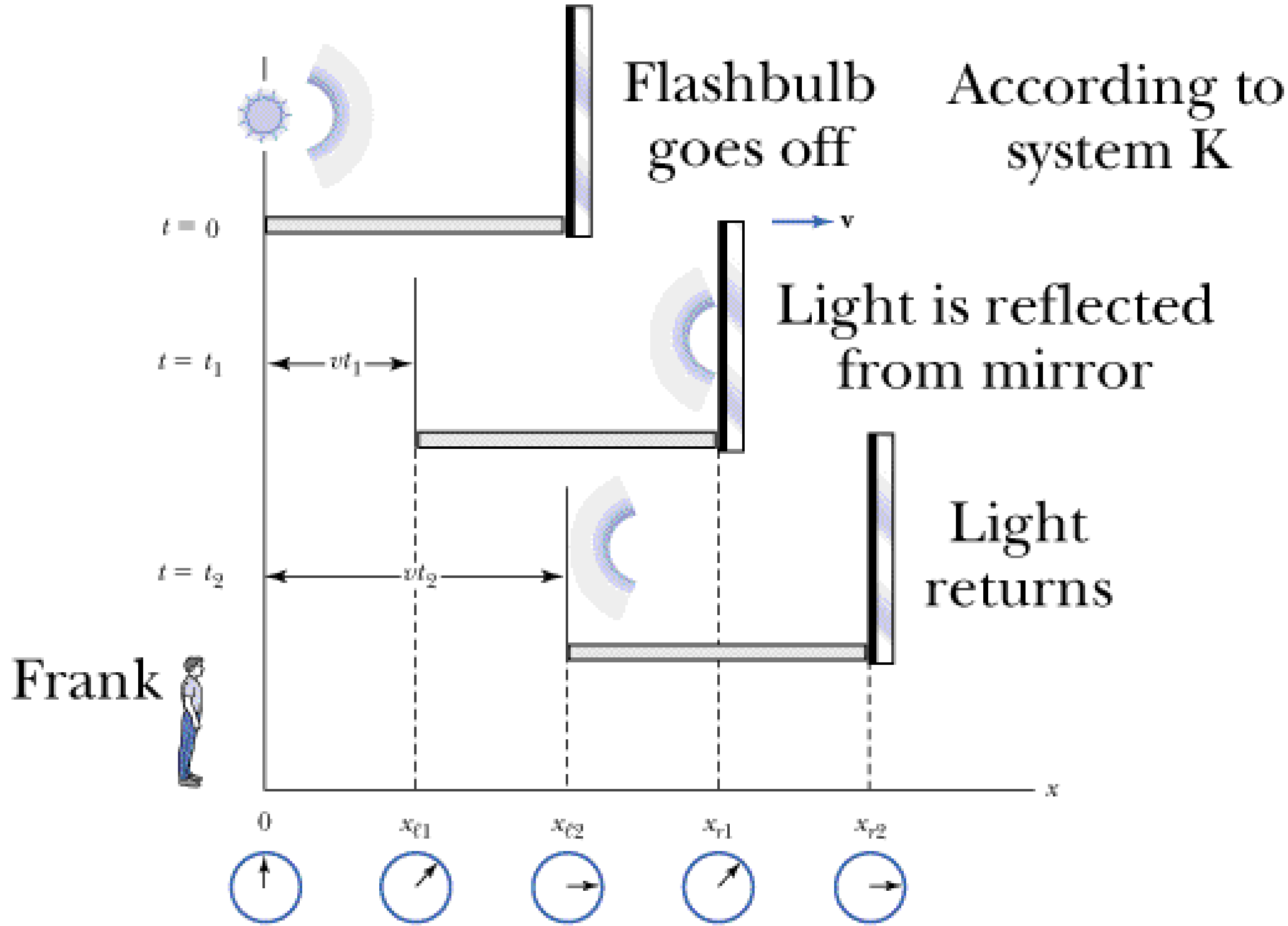

y
Mirror
cT/2
cT/2
L
System K
(at rest)
x
O1
vT/2
vT/2
O2
Frank
Fred
t1 = 0
t = T/2
t2 = T

Example 2.2

It is the year 2050 and the United Nations Space Federation has perfected an antimatter propulsion system. The manned starship is preparing to leave for Alpha Centauri, about 4 lightyears away. Provisions have been placed aboard to allow a trip 16 years' total duration. How fast must the spacecraft travel is the provisions are to last? Neglect acceleration, turnaround, and visiting times, because they are negligible compared to the actual travel times.



(a)


Flashbulb goes off
According to system K
t = 0
v
Light is reflected from mirror
t = t1
vt1
Light returns
t = t2
vt2
Frank
x
0
x_{ℓ1}
x_{ℓ2}
x_{r1}
x_{r2}
(b)

Length Contraction

$$L_0 = \gamma L$$

$$L_0 > L$$

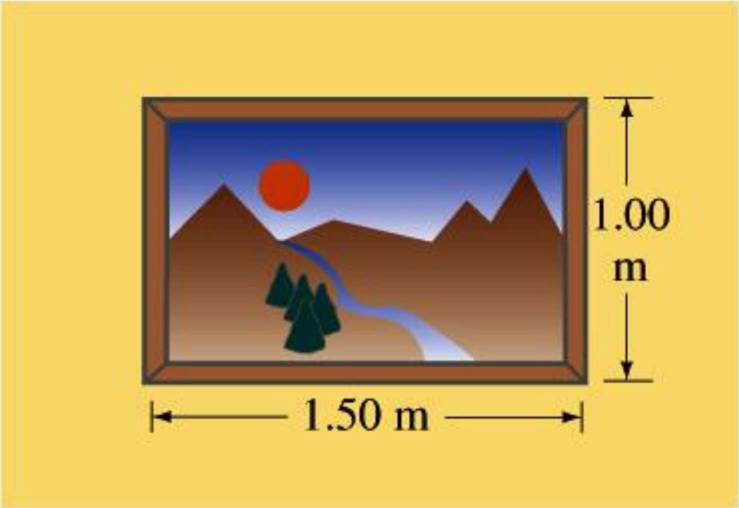
Length Contraction

Length were observer is **moving** relative to the length being measured.

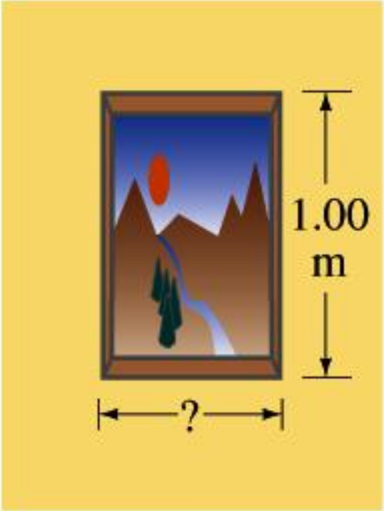
Length were observer is **at rest** relative to the length being measured.

$$L = L_0 \sqrt{1 - v^2/c^2}$$

The length of an object is measured to be shorter when it is moving relative to the observer than when it is at rest.



(a)



(b)

Example 2.3

Consider the solution of Example 2.2 from the standpoint of length contraction.

2.6 Addition of Velocities

$$u_x = \frac{u_x' + v}{1 + (v/c^2)u_x'}$$

$$u_y = \frac{u_y'}{\gamma[1 + (v/c^2)u_x']}$$

$$u_z = \frac{u_z'}{\gamma[1 + (v/c^2)u_x']}$$

Motion on the *x* direction

Notation

- v is the relative velocity of reference frames.
- u is the velocity of an object w.r.t. a particular reference frame.

Example 2.4

The commander of the spaceship just discussed is holding target practice for junior officers by shooting protons at small asteroids and space debris off to the side as the spaceship passes by. What speed will an observer in the space station measure for the protons?

Example 2.5

In 1851, H. L. Fizeau measured the "either drag" coefficient for light passing in opposite directions through flowing water. Let a moving system K be at rest w.r.t. the flowing water and let *v* be the speed of the flowing water w.r.t. a fixed observer in K. The speed of light in the water at rest (i.e., in system K) is *u* , and the speed of light as measured in K is *u*. If the index of refraction of the water is *n*, Fizeau found experimentally that

Example 2.5 (Cont.)

$$u = u \quad + (1 - 1/n^2)v$$

which was in agreement with Fresnel’s prediction. This result was considered confirmation of the either concept. Show that this result can be obtained easily using relativity.

2.7 Experimental Verification

- Muon decay
- Atomic clock measurement
- Velocity addition

Muon Decay

- Cosmic rays enter the upper atmosphere and interact with particles in the upper atmosphere creating π mesons (pions), decay into other particles called muons.
- Obey radioactive decay law

Half life

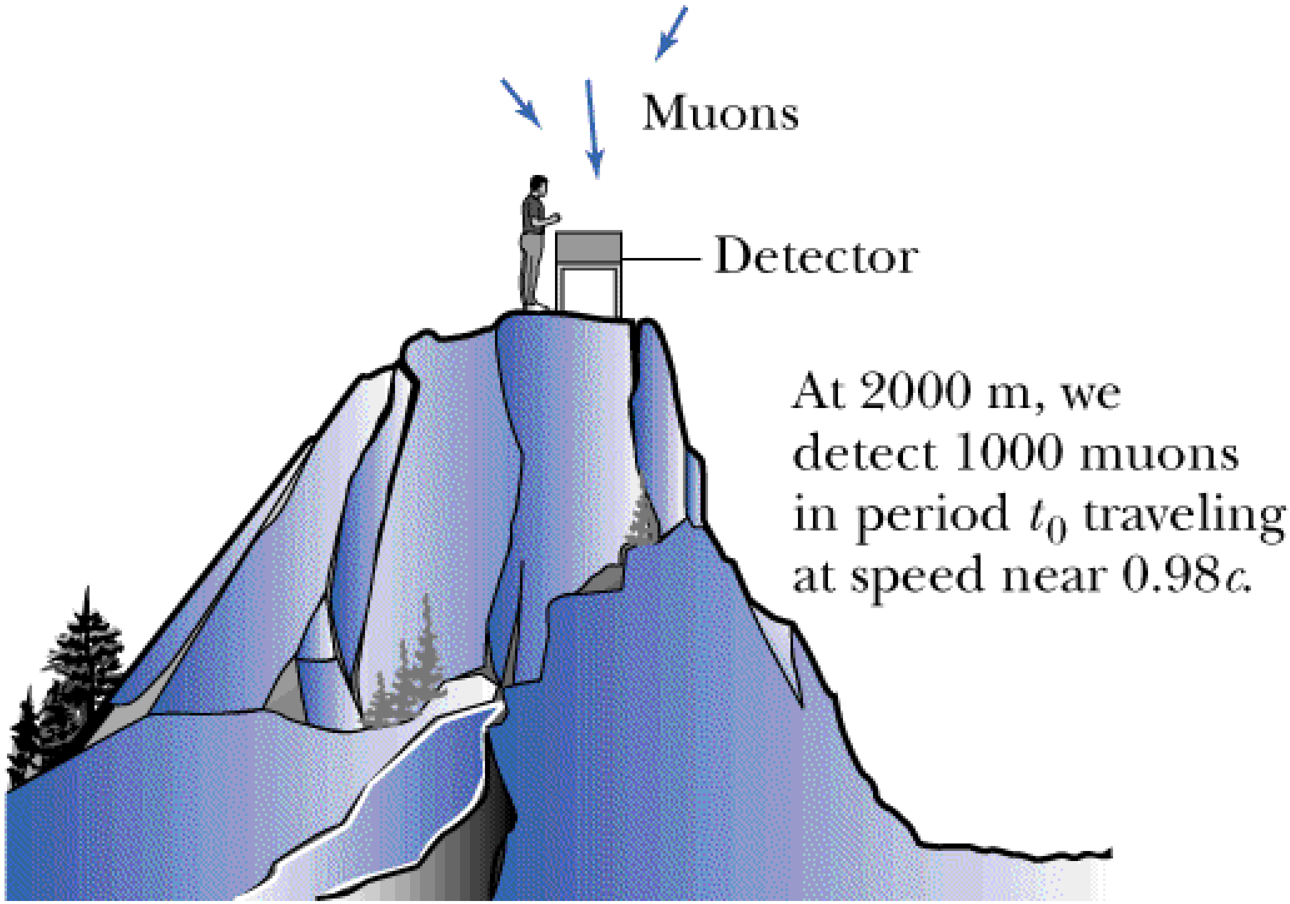
$$N = N_0 e^{-(0.693t/t_{1/2})}$$

No. muons left at time t

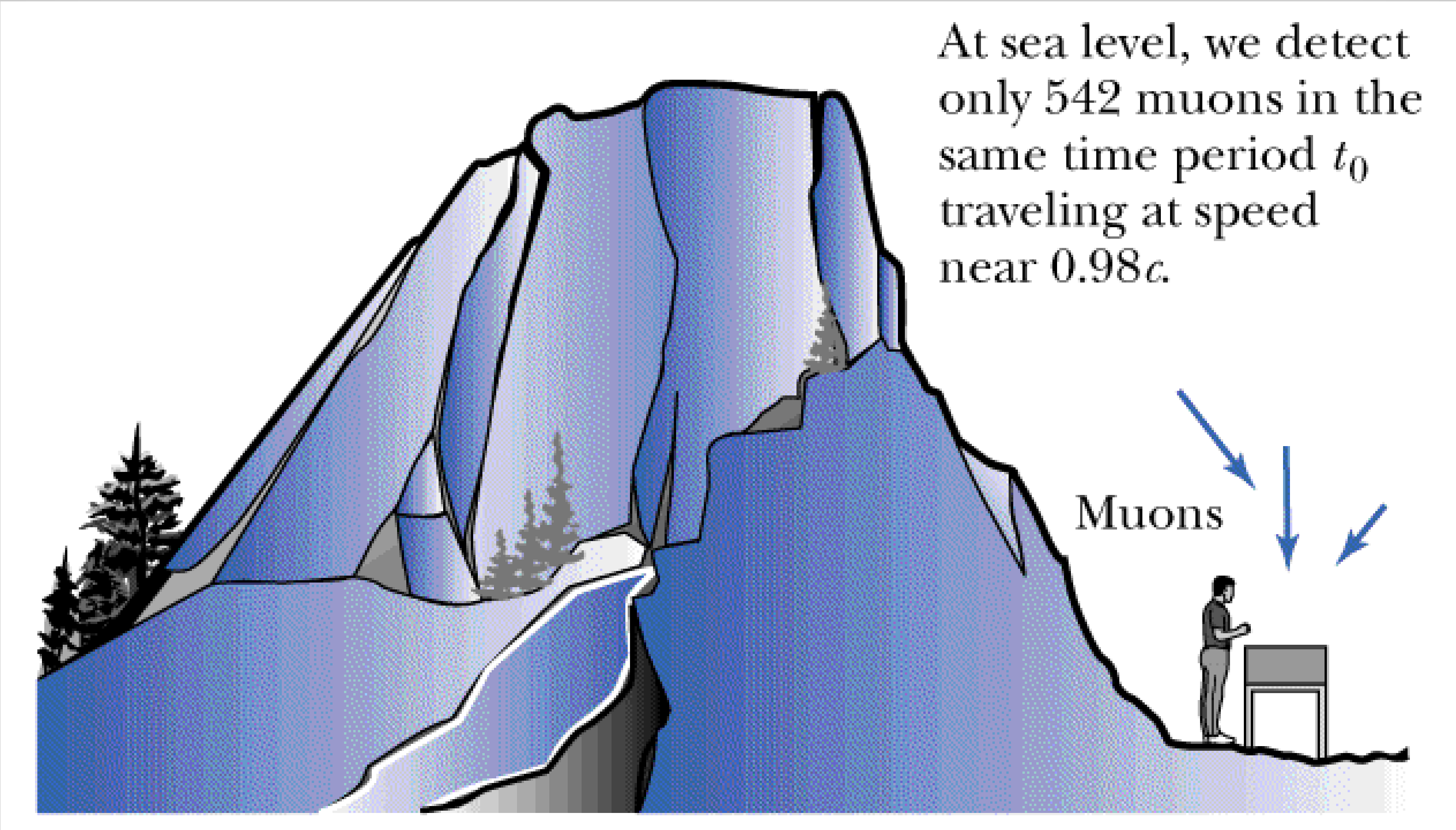
No. muons at $t = 0$

Muon Decay

- $t_{1/2}$ = 1.56 x 10^{-6} s


Muons
Detector
At 2000 m, we
detect 1000 muons
in period t_0 traveling
at speed near 0.98c.

(a)


At sea level, we detect only 542 muons in the same time period t_0 traveling at speed near 0.98c.
Muons

(b)

Atomic Clock Measurement

- 1972 two physicists J.C. Hafele and Richard E. Keating used four ^{133}Cs atomic clocks.
- Flew one clock eastward, one clock westward.
- Two stayed fixed on earth.

Travel	**Predicted**	**Observed**
Eastward	-40 23 ns	-59 10ns
Westward	257 21 ns	273 7 ns

J.C. Hafele and R.E. Keating, *Science* **177**, 166 - 170

Example 2.6

Assuming a jet airplane travels 300 m/s and the circumference of the Earth is about 4 x 10^7 m, calculate the time dilation effect expected for a round-the-world trip exclusive of the Earth's rotation and gravitational correction.

Addition of Velocities

Pion decay experiment at CERN.

2.8 Twin Paradox

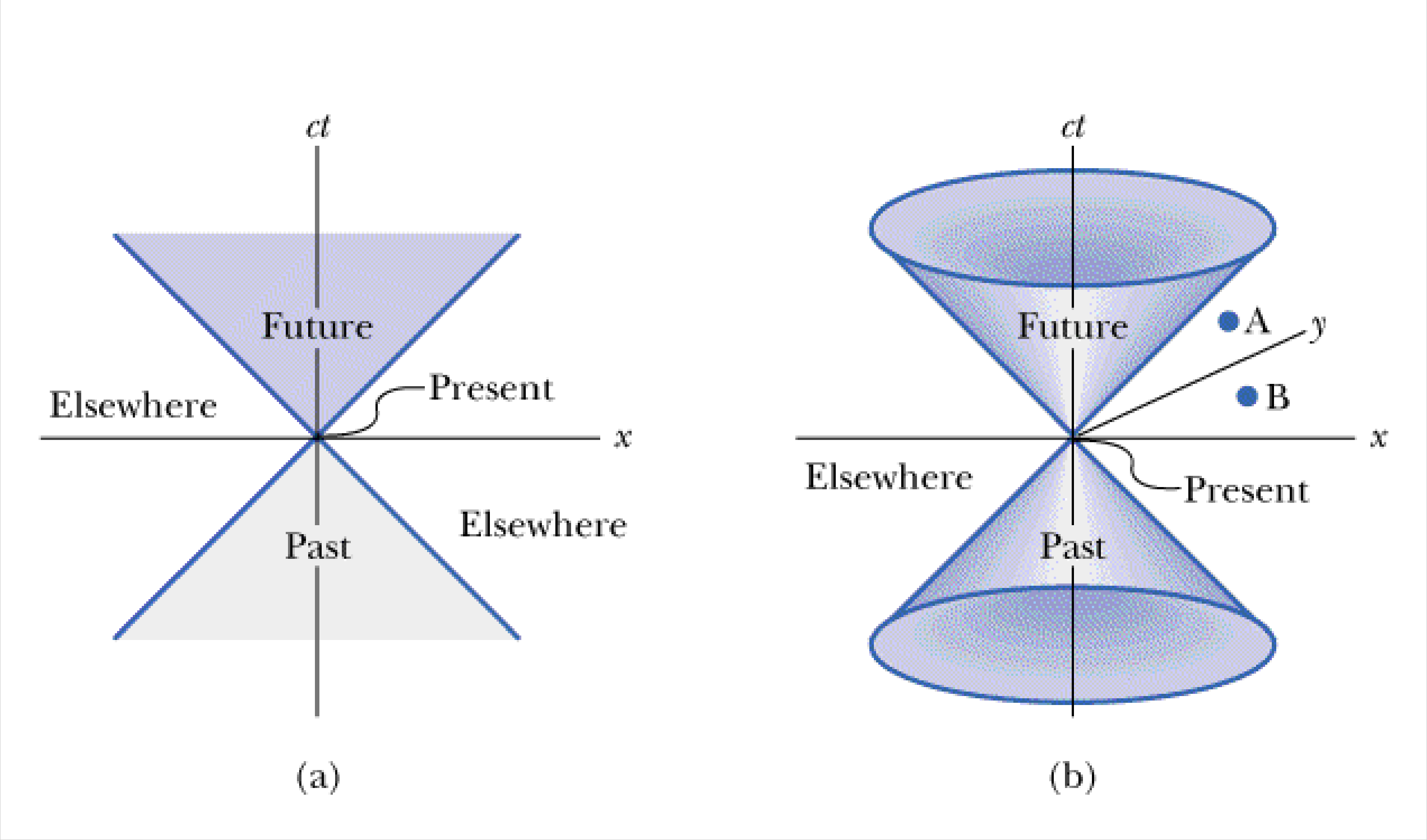
- One twin stays at home.
- One twin travels on a spaceship at very high speeds.
- Relativity says traveling twin will age more slowly.
- But one can say the twin on Earth is traveling w.r.t. the twin in the spaceship and should be the younger.
- This is the paradox. Who is really younger.
- Answer: Traveling twin because of accelerations for the traveling twin—non inertial frame..

TABLE 2.1
Twin Paradox Analysis*

Item	Measured by Frank (remains on Earth)	Measured by Mary (traveling astronaut)
Time of total trip	$T = 2L/v$	$T' = 2L/\gamma v$
Total number of signals sent	$\nu T = 2\nu L/v$	$\nu T' = 2\nu L/\gamma v$
Frequency of signals received at beginning of trip ν'	$\nu\sqrt{\dfrac{1-\beta}{1+\beta}}$	$\nu\sqrt{\dfrac{1-\beta}{1+\beta}}$
Time of detecting Mary's turn-around	$t_1 = L/v + L/c$	$t_1' = L/\gamma v$
Number of signals received at the rate ν'	$\nu' t_1 = \dfrac{\nu L}{v}\sqrt{1-\beta^2}$	$\nu' t_1' = \dfrac{\nu L}{v}(1-\beta)$
Time for remainder of trip	$t_2 = L/v - L/c$	$t_2' = L/\gamma v$
Frequency of signals received at end of trip ν''	$\nu\sqrt{\dfrac{1+\beta}{1-\beta}}$	$\nu\sqrt{\dfrac{1+\beta}{1-\beta}}$
Number of signals received at rate ν''	$\nu'' t_2 = \dfrac{\nu L}{v}\sqrt{1-\beta^2}$	$\nu'' t_2' = \dfrac{\nu L}{v}(1+\beta)$
Total number of signals received	$2\nu L/\gamma v$	$2\nu L/v$
Conclusion as to other twin's measure of time taken	$T' = 2L/\gamma v$	$T = 2L/v$

2.9 Spacetime

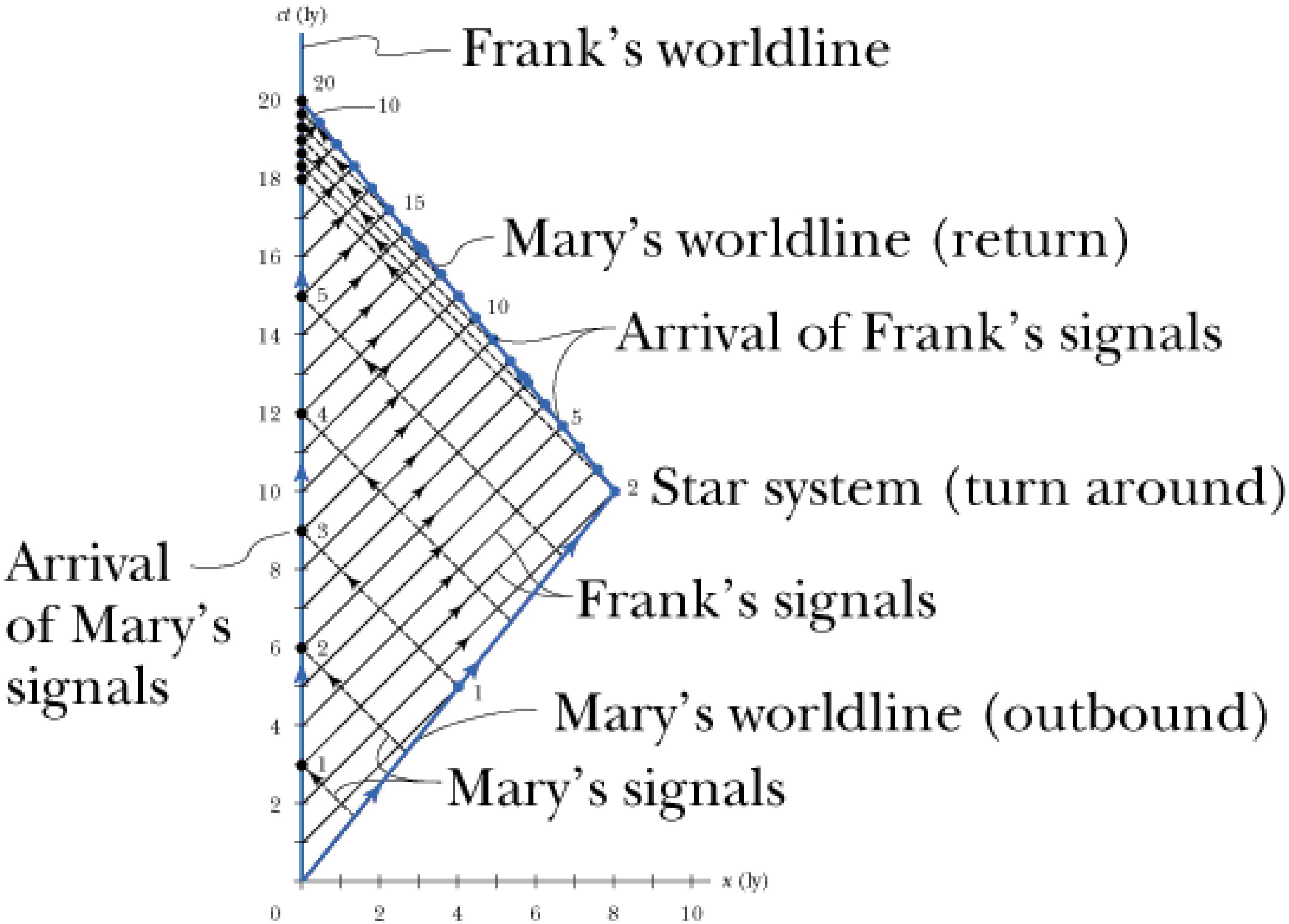
- **Four dimensional spacetime** can be represented by spacetime diagrams.
- First used by Herman Minkowski in 1908 and often called Minkowski diagrams.
- A line connecting two points in spacetime is called a **worldline**.



ct
Future
Present
Elsewhere
x
Past
Elsewhere
(a)
ct
Future
A
y
B
x
Elsewhere
Present
Past
(b)

Example 2.7

Draw the spacetime diagram for the motion of the twins discussed in Section 2.8. Draw light signals being emitted from each twin at annual intervals and count the number of light signals received by each twin from the other.


ct (ly)
Frank's worldline
Mary's worldline (return)
Arrival of Frank's signals
Star system (turn around)
Arrival of Mary's signals
Frank's signals
Mary's worldline (outbound)
Mary's signals
x (ly)

2.10 Doppler Effect

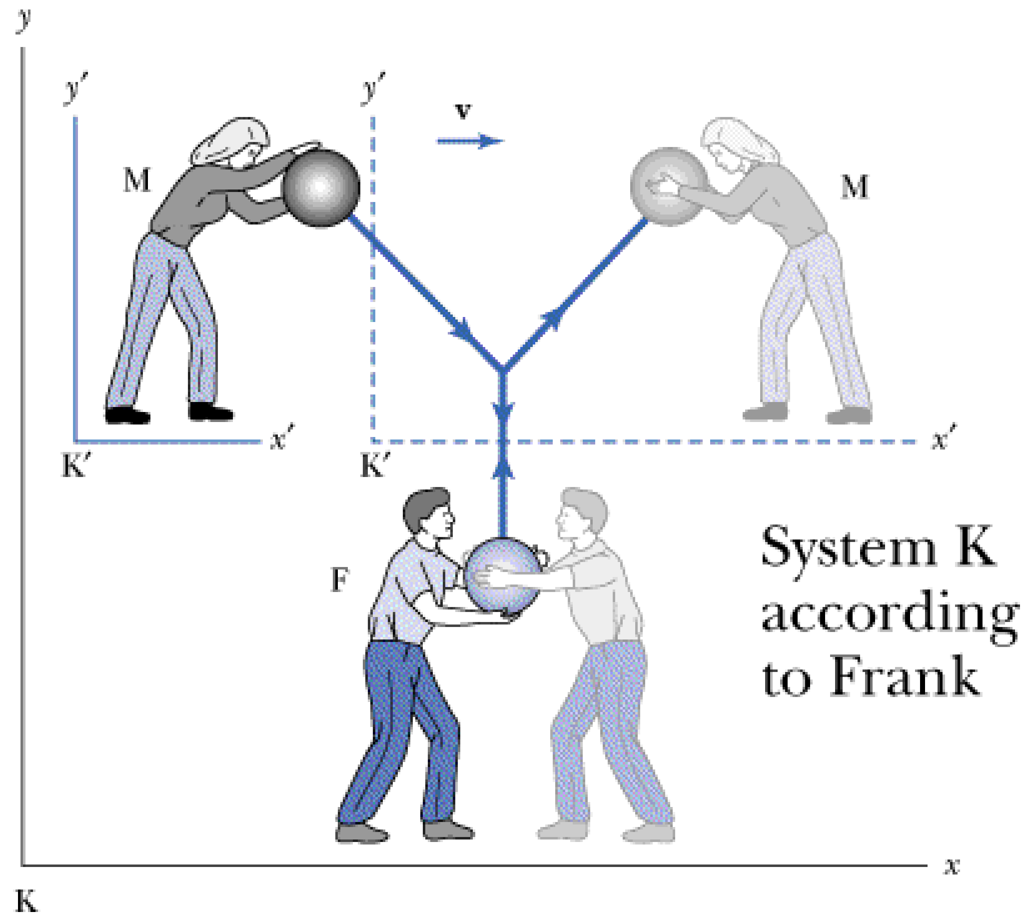
- Difference between relativistic Doppler effect for light and the Doppler effect for sound.
- With sound, the effect depends on which is moving, the source or the receiver.
- With light it is only the relative motion of the source and receiver that counts.

Example 2.8

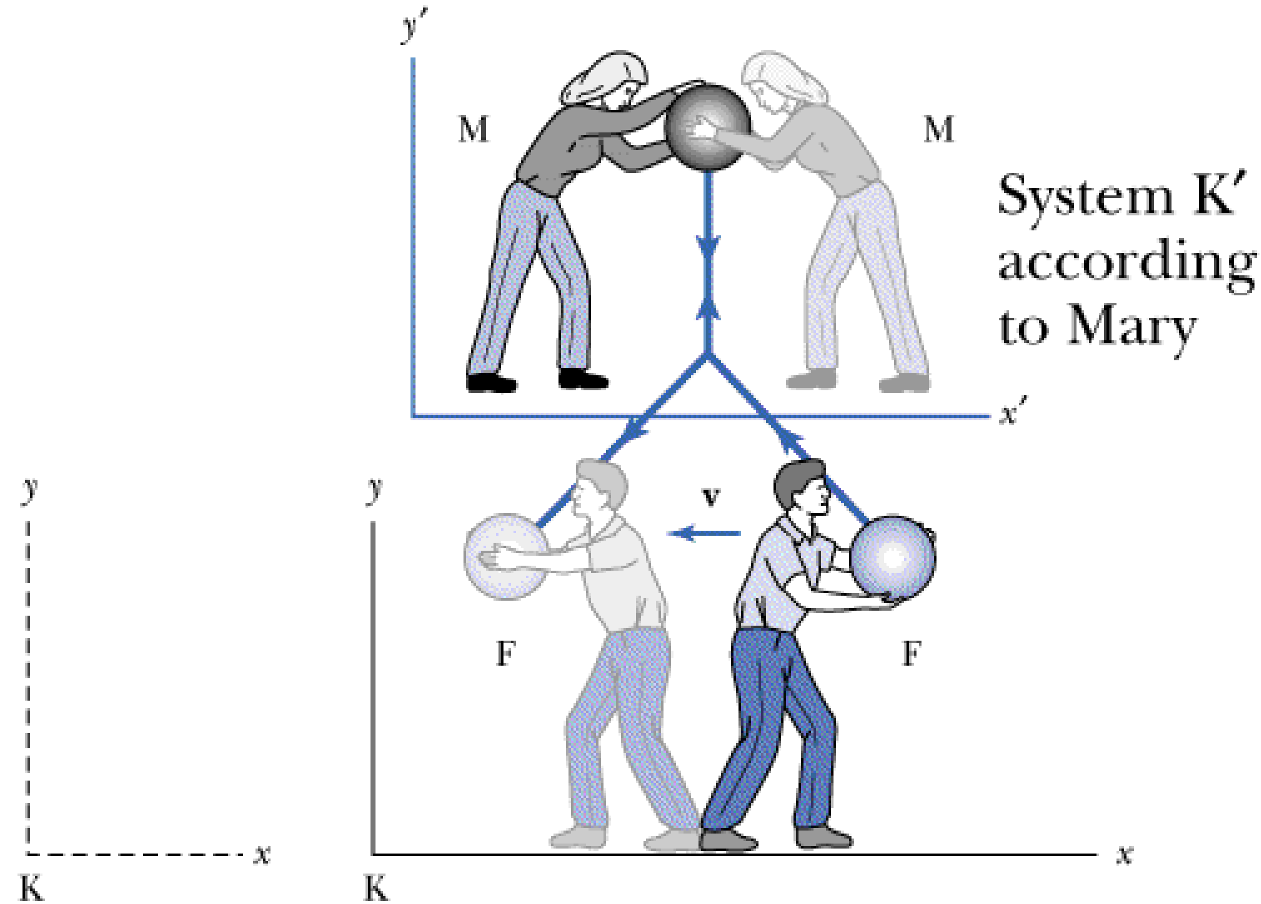
Analyze the light signals sent out by Frank and Mary by using the relativistic Doppler effect.

2.11 Relativistic Momentum

- Know Newton's laws are invariant under a Galilean.
- But, we know that the correct relativistic transformation is the Lorentz transformation.
- Must correct Newton's laws.
- Conservation of linear momentum does not change.



(a)

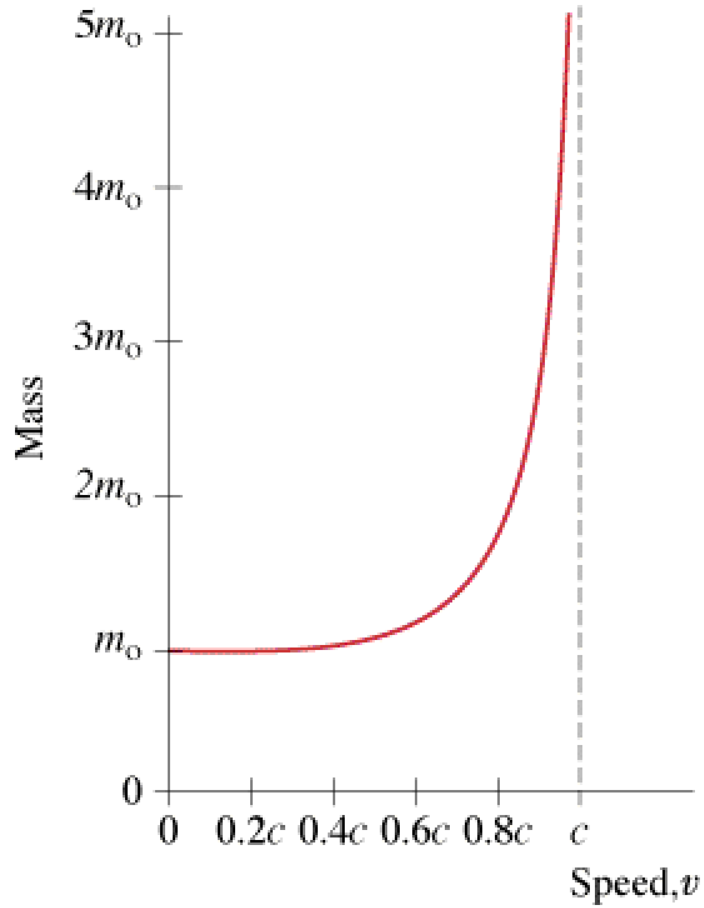


(b)

Relativistic Momentum

$$\mathbf{p} = \gamma m \mathbf{u}$$

Where: $\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}}$


5m0
4m0
3m0
2m0
m0
0
Mass
0
0.2c
0.4c
0.6c
0.8c
c
Speed, v

Example 2.9

Show that linear momentum is conserved for the collision just discussed and shown in the previous figure.

2.12 Relativistic Energy

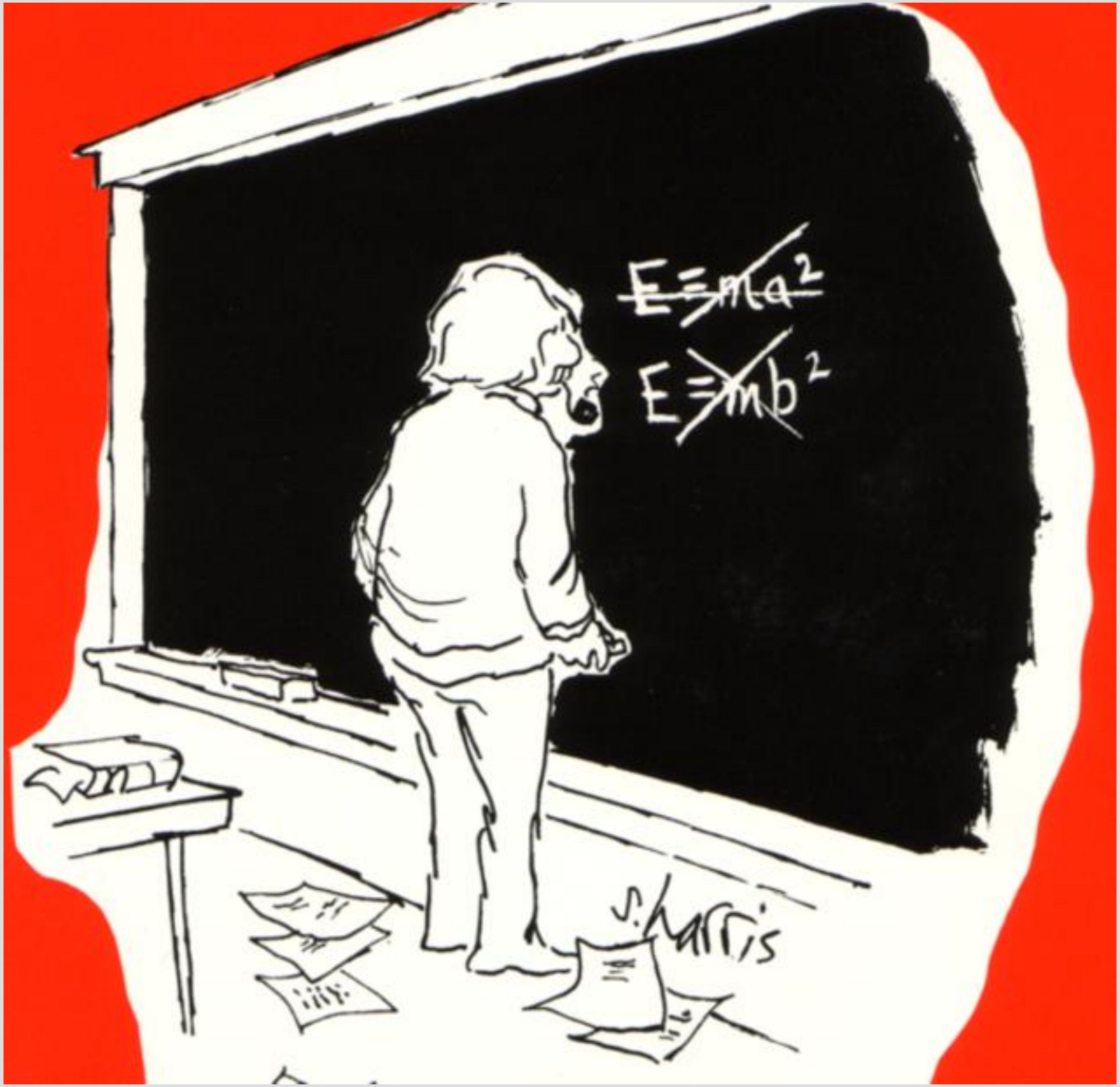
- Consider concepts of energy and force.
- Trying to keep as many concepts from classical physics as necessary.

$$K = mc^2(\gamma - 1)$$

Example 2.10

Electrons in a television set are accelerated by a potential difference of 25,000 volts before striking the screen. Calculate the velocity of the electrons and determine the error in using the classical kinetic energy result.

Total Energy and Rest Energy


E=ma²
E=mb²
S. Harris

Total Energy and Rest Energy

$$E \equiv \gamma mc^2 \text{ (Total Energy)}$$

$$E_0 \equiv mc^2 \text{ (Rest Energy)}$$

Equivalence of Mass and Energy

- ***$E_0 = mc^2$***
- Even when a particle has no velocity and therefore no kinetic energy, it still has energy by virtue of its mass.
- The laws of conservation of energy and conservation of mass must now be combined into one law: ***Law of conservation of mass-energy***.
- **Mass and energy are equivalent**.

Momentum and Mass

Mass measured by an observer **moving** relative to the mass.

Mass measured when object is **at rest** relative to the observer—**rest mass**.

$$m = \frac{m_0}{\sqrt{1 - u^2/c^2}}$$

The mass of an object is measured to increase as its speed increases.

Relationship of Energy and Momentum

- Linear momentum appears to be a more fundamental concept than kinetic.
- No law of conservation of kinetic energy.
- Conservation of linear momentum is inviolate as far as is known.

Relativistic Energy and Momentum

$$E^2 = p^2c^2 + E_0^2$$

$$E^2 = p^2c^2 + m^2c^4$$

Massless Particles

- For a particle having no mass:

$$E = pc$$

- Therefore, for a massless particle:

$$u = c$$

2.13 Computations in Modern Physics

- Common units
  - Energy: Electron volts (eV)
  - Mass:
    - MeV/c^2
    - Unified (atomic) mass unit (u)
  - Momentum: eV/c
- Binding energy

Mole

- 1 mole (mol) of a substance is *defined* as the amount of a substance that contains as many atoms or molecules as there are in 12.00 grams of Carbon 12.
- Mass (^{12}C) = 12 u.
- **1 mol is the number of grams of a substance numerically equal to the molecular mass of a substance.**

Mole

- 1 mol of CO_2 has a mass of [12 + (2x16)] = 44 grams/mol.

- $$n \text{ (mol)} = \frac{\text{mass (grams)}}{\text{molecular mass (g/mol)}}$$

- The number of mols in 132 grams of CO_2 is:

- $$n = \frac{132\text{g}}{44\text{g/mol}} = 3.0 \text{ mol}$$

Ideal gas Law in Terms of Molecules: Avogadro's Number

- Amedeo Avogadro (1776 – 1856)
- Avogadro's hypothesis states: ***equal volumes of gas at the same pressure and temperature contain equal numbers of molecules.***
- Avogadro's number:

$$N_A = 6.02 \times 10^{23} \text{ molecules/mole.}$$

- That is, Avogadro was the first to clearly realize that the volume of a gas depends on the number of molecules it contains.
- Or, 1 mole of any substance contains N_A numbers of molecules (or atoms as the case may be).

In Other Words

A **mole** of oxygen (32 g) is **16 times as massive as a mole of hydrogen** gas (2 g), but is made of **molecules that are 16 times as massive as hydrogen.** Therefore, one mole of oxygen contains the same number of molecules as one mole of hydrogen.

Calculation of Avogadro's Number

1 atom of mass *m* of ^{12}C = 1.9926 x 10^{-26} kg/molecule

The atomic mass of ^{12}C = 12 g/mol (12 x 10^{-3} kg/mol)

$$\frac{12 \times 10^{-3} \text{ kg/mol}}{1.9926 \times 10^{-26} \text{ kg/molecule}} = 6.0214 \times 10^{23} \text{ molecules/mol}$$

= Avogadro's Number (N_A)

Example 2.11

A 2-GeV proton hits another 2-GeV proton in a head-on collision. (a) Calculate v, β, p, K, and E for each of the protons. (b) What happens to the kinetic energy?

Binding Energy

- The potential energy associated with the force keeping a system together is called the **binding energy** E_{B}.
- The binding energy of a system is the work required to pull the particles out of the system into separate, free particles at rest

Binding Energy

$$E_B = \sum m_i c^2 - M_{\text{bound system}} c^2$$

Example 2.12

What is the minimum kinetic energy the protons must have in the head-on collision of Equation (2.78), $p + p \longrightarrow \pi^+ + d$, in order to produce the positively charged pion and deuteron? The mass of the π^+ is 139.6 MeV/c^2.

Example 2.13

The atomic mass of the ^{4}He atom is 4.002603 u. Find the energy of the ^{4}He nucleus.

Example 2.14

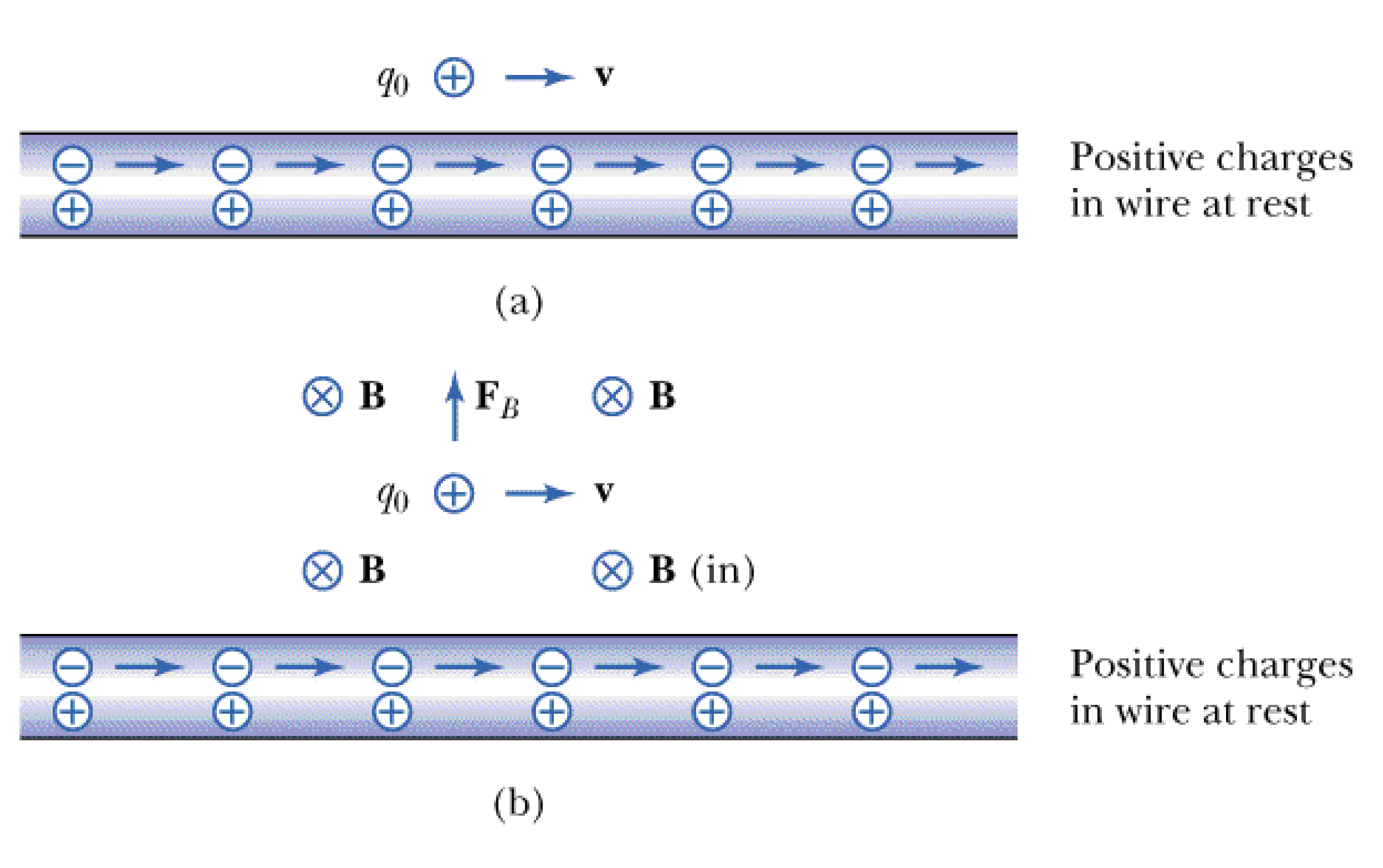
The molecular binding energy is called the *dissociation energy*. It is the energy required to separate the atoms in a molecule. The dissociation energy of the NaCl molecule is 4.24 eV. Determine the fractional mass increase of the Na and Cl atoms separate from NaCl. What is the mass increase for a mole of NaCl?

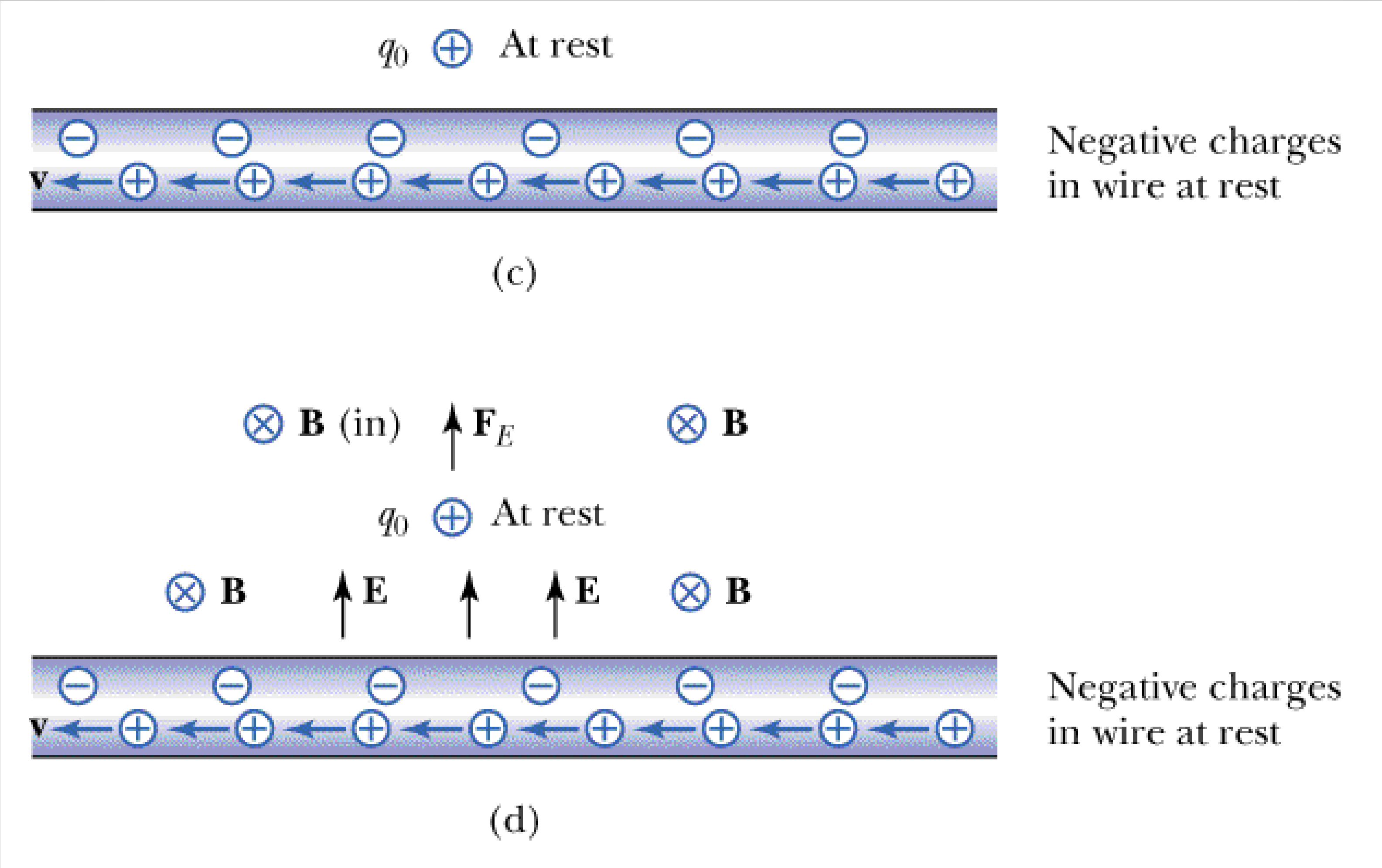
Example 2.15

A positively charged Σ^+ particle produced in a particle physics experiment decays very quickly into a neutron and positively charged pion before either its energy or momentum can be measured. The neutron and the pion are observed to move in the same direction as the Σ^+ was originally moving, with momenta of 4702 MeV/*c* and 169 MeV/*c*, respectively. What was the kinetic energy of the Σ^+ and its mass?

2.14 Electromagnetism and Relativity

- Einstein was convinced that magnetic fields were nothing other than electric fields observed in another inertial frame.
- That is the key to electromagnetism and relativity.


q0
v
Positive charges
in wire at rest
(a)
B
F_B
B
q0
v
B
B (in)
Positive charges
in wire at rest
(b)



q_0 At rest
Negative charges in wire at rest
v
(c)
B (in)
F_E
B
q_0 At rest
B
E
E
B
Negative charges in wire at rest
v
(d)

Homework Problem 13

Two events occur in an inertial system K as follows:

Event 1: $x_1 = a$, $t_1 = 2a/c$, $y_1 = 0$, $z_1 = 0$

Event 2: $x_2 = 2a$, $t_2 = 3a/2c$, $y_1 = 0$, $z_1 = 0$

In what frame K will these events appear to occur at the same time? Describe the motion of the system K .

Homework Problem 19

A rocket ship carrying passengers blasts off to go from New York to Los Angles, a distance of 5000 km. How fast must the rocket ship go to have its length shortened by 1%?

Homework Problem 22

The Apollo astronauts returned from the moon under Earth's gravitational force and reached speeds of almost 25,000 mi/h with respect to the Earth. Assuming (incorrectly) they had this speed for the entire trip from the moon to the Earth, what was the time difference for the trip between their clocks and clocks on Earth?

Homework Problem 31

A spaceship is moving at a speed of 0.80 c away from an observer at rest. A boy in the spaceship shoots a proton gun with protons having a speed of 0.70 c. What is the speed of the protons measured by the observer at rest when the gun is shot (a) away from the observer and (b) toward the observer?

Homework Problem 59

A particle having a speed of 0.92 *c* has a momentum of 10^{-16} kg•m/s. What is its mass?

Homework Problem 67

Calculate the momentum, kinetic energy, and total energy of an electron traveling at a speed of (a) 0.01 c, (b) 0.1 c, and (c) 0.9 c.

Homework Problem 80

A free neutron is an unstable particle and beta decays into a proton with the emission of an electron. How much kinetic energy is available in the decay?