Thornton and Rex Chap 2: Special Theory of Relativity

HW #4 Solutions (Due October 15, 2012)

19. (a) With a contraction of 1%, $L/L_0 = 0.99 = \sqrt{1-v^2/c^2}$. Thus $1-\beta^2 = (0.99)^2 = 0.9801$.
Solving for β, we find $\beta = 0.14$ or $v = 0.14c$.
(b) The time for the trip in the Earth-based frame is
$\Delta t = \dfrac{d}{v} = \dfrac{5.00\times 10^6\,\text{m}}{0.14\times 3.00\times 10^8\,\text{m/s}} = 1.19\times 10^{-1}\,\text{s}$. With the relativistic factor $\gamma = 1.01$
(corresponding to a 1% shortening of the ship's length), the elapsed time on the rocket ship is 1% less than the Earth-based time, or a difference of
$(0.01)1.2\times 10^{-1}\,\text{s} = 1.2\times 10^{-3}\,\text{s}$.

27. Spacetime invariant (see Section 2.9): $c^2\Delta t^2 - \Delta x^2 = c^2\Delta t'^2 - \Delta x'^2$. We know $\Delta x = 4$ km, $\Delta t = 0$, and $\Delta x' = 5$ km. Thus $\Delta t'^2 = \dfrac{\Delta x'^2 - \Delta x^2}{c^2} = \dfrac{(5000\text{ m})^2 - (4000\text{ m})^2}{(3.00\times 10^8\text{ m/s})^2} = 1.0\times 10^{-10}\text{ s}^2$

and $\Delta t' = 1.0\times 10^{-5}$ s.

57. $\vec{p} = \gamma m\vec{v} = \dfrac{m\vec{v}}{\sqrt{1-v^2/c^2}}$ and $\vec{F} = \dfrac{d\vec{p}}{dt}$. The momentum is the product of two factors that contain the velocity, so we apply the product rule for derivatives:

$$
\begin{aligned}
\vec{F} &= m\frac{d}{dt}\left[\frac{m\vec{v}}{\sqrt{1-v^2/c^2}}\right] \\
&= m\left[\frac{d\vec{v}/dt}{\sqrt{1-v^2/c^2}} + \vec{v}\frac{d}{dt}\left(\frac{1}{\sqrt{1-v^2/c^2}}\right)\right] \\
&= \gamma m\vec{a} + m\vec{v}\left(-\frac{1}{2}\right)\left(-\frac{2v}{c^2}\right)\gamma^3\frac{dv}{dt} \\
&= \gamma m\vec{a} + \gamma^3 m\vec{a}\left(\frac{v^2}{c^2}\right) \\
&= \gamma^3 m\vec{a}\left[1 - \frac{v^2}{c^2} + \frac{v^2}{c^2}\right] \\
&= \gamma^3 m\vec{a}
\end{aligned}
$$

67. (a) $p = \gamma m u = \dfrac{\left(511\ \text{keV}/c^2\right)\left(0.020c\right)}{\sqrt{1-0.020^2}} = 10.22\ \text{keV}/c$;

$$E = \gamma mc^2 = \frac{\left(511\ \text{keV}/c^2\right)\left(c^2\right)}{\sqrt{1-0.02^2}} = 511.102\ \text{keV};$$

$$K = E - E_0 = 511.102\ \text{keV} - 511.00\ \text{keV} = 102\ \text{eV}$$

The results for (b) and (c) follow with similar computations and are tabulated:

β	p (keV/c)	E (keV)	K (keV)
0.20	104.3	521.5	10.5
0.90	1055	1172	661